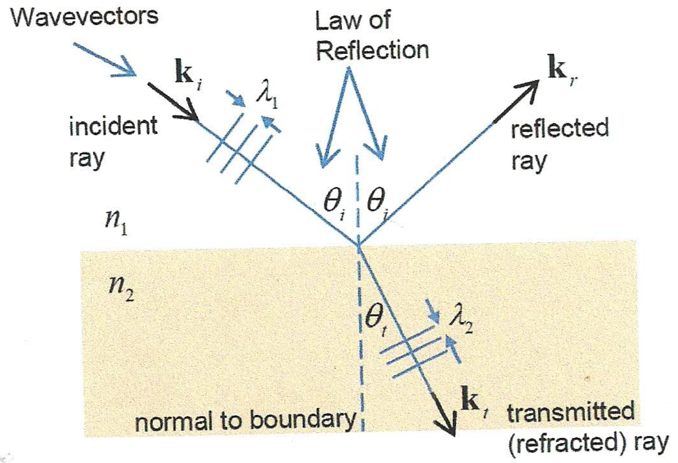
WRITE ON THIS PAPER UNLESS TOLD OTHERWISE. CLEARLY SHOW ALL WORKINGS! PAY ATTENTION TO NEATNESS AND ORGANIZATION. HAND IT IN ON TIME. HAVE A GO EVEN IF AT FIRST YOU CAN'T SPOT THE ANSWER!

NAME: Dr French SET: .... DATE: ....

Waves worksheet! Recall the following relationships:

- The meaning of amplitude A, wavelength λ, and wave speed c as defined in the **Basic waves anatomy handout.**
- Wave period = *time* T between wave crests
- Frequency (the number of waves per second, measured in Hertz, Hz). $f = 1/T$
- Wave speed equation: $c = f\lambda$
- For **reflection**, angle *of incidence* = *angle of reflection* (both from surface normal)
- *Refractive index* $n = \dfrac{\text{speed of light in a vaccum}}{\text{speed of light in a medium}}$ ($n = 1$ for vacuum, about 1.0 for air, about 1.5 for glass)
- For **refraction** of waves at a boundary between to mediums of different wave speeds we have Snell's Law $\dfrac{\sin\theta_1}{c_1} = \dfrac{\sin\theta_2}{c_2}$, which for light (i.e. using refractive index) is $n_1 \sin\theta_1 = n_2 \sin\theta_2$. As for reflection, the angles are measured from the surface normal.
- The number of waves per second entering a boundary must equal the number of waves exiting i.e. frequency is conserved. Hence if wave speed changes, so does wavelength in the same proportion.

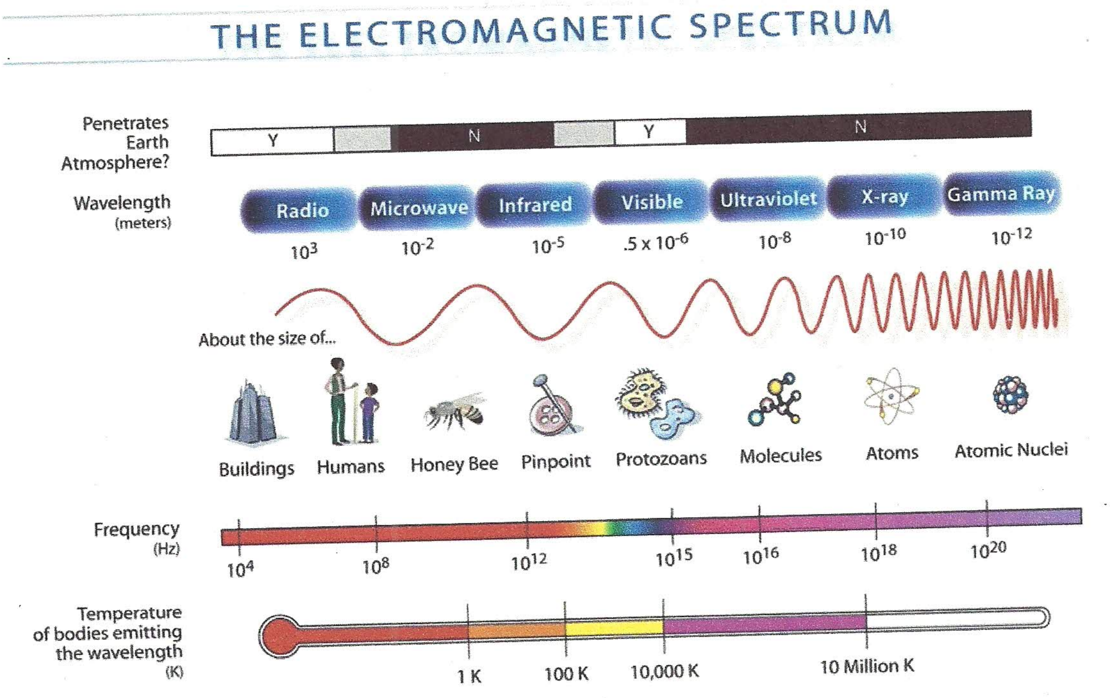
$$f = \frac{c}{\lambda} \quad \therefore \frac{c_1}{\lambda_1} = \frac{c_2}{\lambda_2}$$



1 mile = 1609.34m   1 hour = 3600s $1\text{ms} = 10^{-3}\text{s}$
$1\mu\text{s} = 10^{-6}\text{s}$ $1\text{ns} = 10^{-9}\text{s}$ $1\text{kHz} = 10^3\text{Hz}$
$1\text{MHz} = 10^6\text{Hz}$ $1\text{GHz} = 10^9\text{Hz}$ $1\text{THz} = 10^{12}\text{Hz}$

Speed of sound in air is about 340 m/s
Speed of sound in water is about 1500 m/s
Speed of sound in rock is about 5000 m/s
Speed of light in a vacuum is $c = 2.998 \times 10^8$ m/s

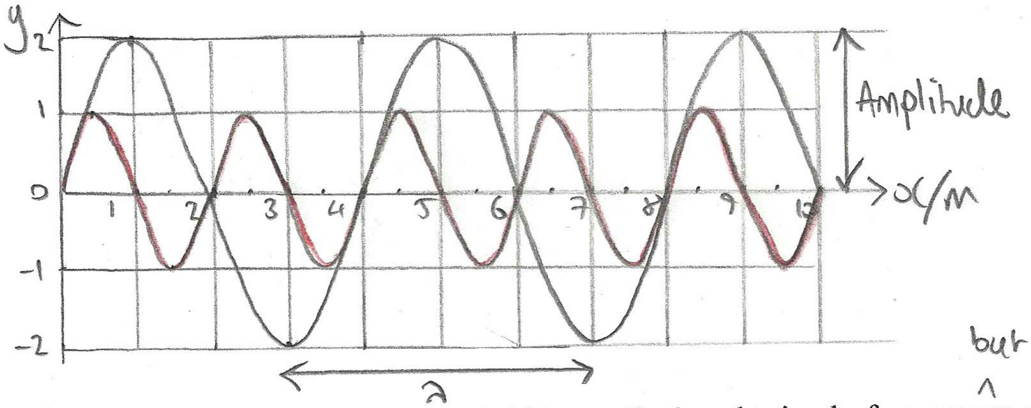
All electromagnetic waves travel at the speed of light.



Question set A

$\lambda = 4.0\text{m}$

(i) A sinusoidal wave has amplitude 2.0 units and wavelength ~~3.0~~m. Sketch a y vs x snapshot of the wave below, with *two full oscillations* shown. Annotate clearly what corresponds to the amplitude and wavelength.



but the same wave speed

(ii) Overlay on the graph above a sinusoid with half the amplitude and twice the frequency as above.

ie, half the wavelength

(iii) A sound wave of frequency 440Hz travels through air at $c = 340\text{m/s}$.

(a) Calculate the period /s

$T = \frac{1}{f} = \frac{1}{440}\text{ s} = \boxed{2.27\times10^{-3}\text{s}}$

(2.27ms)

(b) Calculate the wavelength /m

$c = f\lambda \quad \therefore \lambda = \frac{c}{f} = \frac{340\text{ m/s}}{440\text{ Hz}} = \boxed{0.773\text{ m}}$

(iv) Microwaves are electromagnetic waves with a wavelength of about ~~1~~ 10 cm. Calculate their frequency in GHz, to 1.s.f.

$c = f\lambda \quad \therefore f = \frac{c}{\lambda} = \frac{2.998\times10^{8}\text{ m/s}}{0.1\text{ m}} = 2.998\times10^{9}\text{ Hz}$

$\approx \boxed{3\text{ GHz}}$

["S band". Many radars, and mobile phones work at this frequency]

(v) Dolphins communicate[1] socially with sounds between 0.2 and 40kHz. Higher frequency clicks (40-150kHz) are used for echo-location. Assuming the speed of sound in water is 1500m/s

(a) Calculate the wavelength of a 40kHz 'squeak' in metres

$c = f\lambda \quad \therefore \lambda = \frac{c}{f} = \frac{1500\text{ m/s}}{40\times10^{3}\text{ Hz}} = \boxed{0.038\text{ m}}$

(b) Calculate the period of a 150kHz click / μs

$T = \frac{1}{f} = \frac{1}{150\times10^{3}}\text{ s}$

$= 6.7\times10^{-6}\text{s} = \boxed{6.7\,\mu\text{s}}$

(c) Calculate the wavelength /cm of a 150kHz click

$\lambda = \frac{c}{f} = \frac{1500\text{m}}{150\times10^{3}\text{ Hz}} = 0.01\text{ m} = \boxed{1\text{ cm}}$

[1] https://seaworld.org/en/animal-info/animal-infobooks/bottlenose-dolphins/communication-and-echolocation

Question set B F#

A octave below $\xleftarrow{\div 2}$ A $\xrightarrow{\times 2}$ A next octave

(i) In the *equal temperament* system of tuning, the frequency of *semitones* in an *octave* C,C#,D,Eb,E,F,~~G#~~,G#,G,Ab,A,Bb,B,C.... form a *geometric series*. This means the frequency of each semitone is the same *ratio* as the previous. This ratio is:

$(2^{\frac{1}{12}})^{12} = 2$

$$\frac{f_{n+1}}{f_n} = 2^{\frac{1}{12}} \approx 1.05946$$ i.e. each semitone is about 6% higher in frequency

$2^{\frac{1}{12}} \equiv \sqrt[12]{2}$

'Concert A' is often defined as 440Hz. This means:

Bb is $f_{Bb} = 440 \times 2^{\frac{1}{12}} = 466.2\text{Hz}$

B is $f_B = 440 \times 2^{\frac{1}{12}} \times 2^{\frac{1}{12}} = 440 \times 2^{\frac{2}{12}} = 493.9\text{Hz}$

E is $f_E = 440 \times 2^{\frac{-5}{12}} = 329.6\text{Hz}$

EQUAL TEMPERED SCALE

C $\times 2^{\frac{1}{12}}$
C#/Db $\times 2^{\frac{1}{12}}$
D $\times 2^{\frac{1}{12}}$
Eb/D# $\times 2^{\frac{1}{12}}$
E $\times 2^{\frac{1}{12}}$
F $\times 2^{\frac{1}{12}}$
F#/Gb $\times 2^{\frac{1}{12}}$
G $\times 2^{\frac{1}{12}}$
Ab/G# $\times 2^{\frac{1}{12}}$
A $\times 2^{\frac{1}{12}}$
Bb/A# $\times 2^{\frac{1}{12}}$
B $\times 2^{\frac{1}{12}}$
C $\times 2^{\frac{1}{12}}$

(a) Calculate the frequencies (in Hz) of:

A an octave below concert A $\boxed{220\text{ Hz}}$ (i.e. $440 \times 2^{-12/12}$)

C above concert A $440 \times 2^{3/12} = \boxed{523.3\text{ Hz}}$

C# below concert A $440 \times 2^{-8/12} = \boxed{277.2\text{Hz}}$

Eb an octave *higher* than the octave of concert A $440 \times 2^{\frac{6}{12}} = \boxed{622.3\text{ Hz}}$
($\sqrt{2}\times$higher)

(b) *Baroque pitch* defines a 'Concert A' to be **415Hz**. Fill in the table below for the frequencies of notes, and also the wavelength of these sounds in air. Assume the speed of sound is 340m/s.

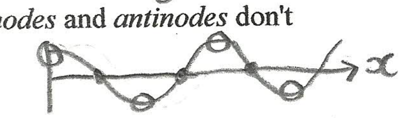
Note	Frequency f (i.e. pitch) /Hz	Wavelength in air λ /m
F	$415 \times 2^{\frac{-4}{12}} =$ 329.4	340/329.4 = 1.03
~~G#~~ F#	$415 \times 2^{-3/12} = 349.0$	340/349.0 = 0.97
G	$415 \times 2^{-2/12} = 369.7$	340/369.7 = 0.92
Ab	$415 \times 2^{-\frac{1}{12}} =$ 391.7	340/391.7 = 0.87
A	415	$340\text{ms}^{-1}/415\text{Hz} =$ ~~0.819~~ 0.82
Bb	$415 \times 2^{\frac{1}{12}} =$ 439.7	340/439.7 = 0.77
B	$415 \times 2^{2/12} = 465.8$	340/465.8 = 0.73
C	$415 \times 2^{3/12} = 493.5$	340/493.5 = 0.69
C#	$415 \times 2^{4/12} = 522.9$	340/522.9 = 0.65
D	$415 \times 2^{\frac{5}{12}} =$ 554.0	340/554.0 = 0.61

$\lambda = 340/f$

• Nodes

o Antinodes

(ii) Sound played through a *Kundt's* tube will form *standing waves*, i.e. where the *nodes* and *antinodes* don't move. This often is a resonance situation, i.e. which produces the loudest waves.



The Kundt's tube is open at one end and driven at the other end, which means standing waves of air pressure must have a node at the open end, and an antinode at the driven (loudspeaker) end. $L = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4} \ldots$

This means the length of the tube must equal an odd number of quarter wavelengths at resonance.

(a) Using the box below, sketch the variation of pressure vs length in the tube for the lowest frequency standing wave (we call this the 'fundamental')

FUNDAMENTAL

↑ Pressure above atmospheric ΔP

ie L is o •

Pressure wave oscillates in time within this envelope

(b) Use the boxes below to sketch the pressure variation vs length for the first and second *harmonics*.

ie L is o • o • ← 3 spacings

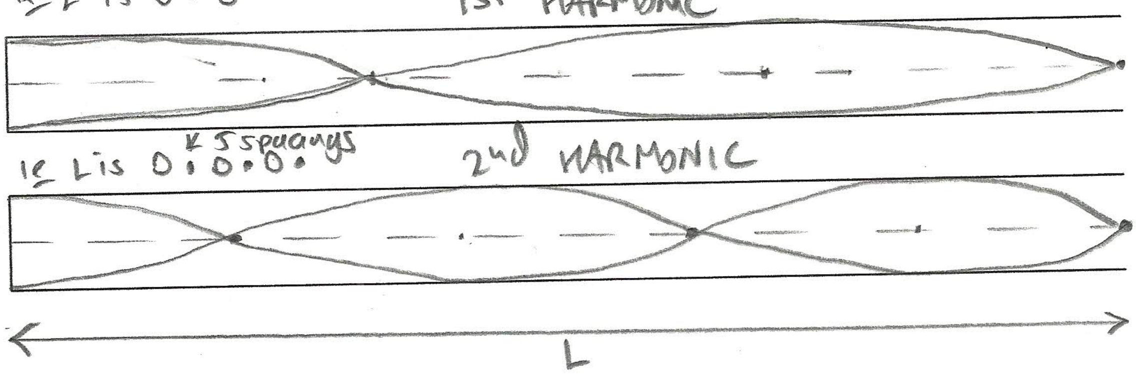
1st HARMONIC

ΔP ↑

ie L is o • o • o • ← 5 spacings

2nd HARMONIC

ΔP ↑



In reality the nodes will be slightly beyond the open end due to END CORRECTION

(c) If the Kundt's tube is 1.5m long and the speed of sound in air is 340m/s, calculate the frequencies (in Hz) of the fundamental and the first and second harmonics.

$L \to 0.6r$

↑ tube radius

$c = f\lambda$

$L = (2n-1)\frac{\lambda}{4}$

$n = 1, 2, 3, 4 \ldots$

ie $2n-1$ are the odd integers $1, 3, 5, 7 \ldots$

So $\lambda = \frac{4L}{2n-1}$

$f = \frac{c}{\lambda}$

$\therefore f = \frac{c}{\frac{4L}{2n-1}}$

$\therefore \boxed{f_n = (2n-1)\frac{c}{4L}}$

Fundamental $\therefore f_1 = 1 \times \frac{340}{4 \times 1.5} = \boxed{56.7\text{ Hz}}$

1st harmonic $f_2 = 3 \times \frac{340}{4 \times 1.5} = \boxed{170\text{ Hz}}$

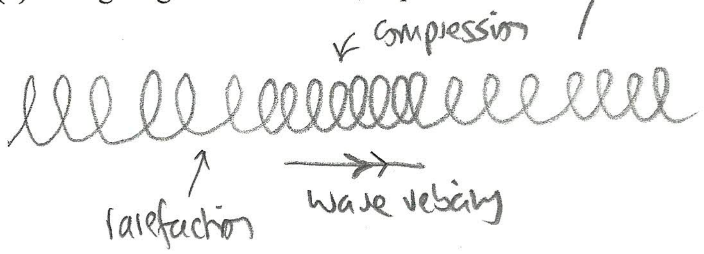
2nd harmonic $f_3 = 5 \times \frac{340}{4 \times 1.5} = \boxed{283\text{ Hz}}$

This explains why you need a long tube (think organ or alpine horn) to produce really low bass notes!

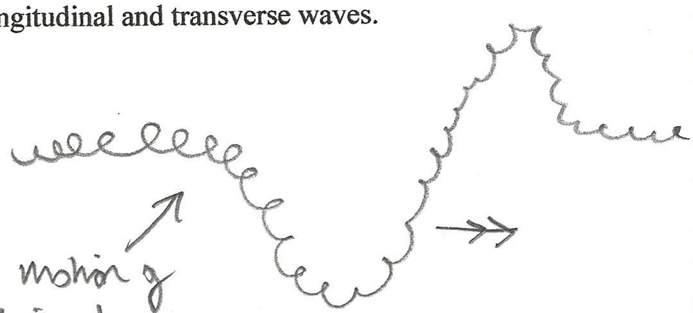
ie motion of coils is $\parallel$ to direction of wave propagation

Question set C

(i) (a) Using diagrams of slinkies, explain the difference between longitudinal and transverse waves.



LONGITUDINAL



ie motion of coils is $\perp$ to direction of wave propagation

TRANSVERSE

(b) Fill in the following table with YES or NO answers

Wave	Longitudinal?	Transverse?
Sound wave in air	YES	NO
* X-ray	NO	YES
Earthquake shear wave	NO	YES
* Radio signal	NO	YES
Earthquake pressure-wave	YES	NO
Surface water wave	NO	YES
Whale song in the ocean	YES	NO
* Sunlight	NO	YES
Standing waves in the air column of a musical wind instrument like a flute *	YES	NO
Vibration of a guitar string	NO	YES

* Electro-magnetic waves.

* Assume either molecule displacement OR air pressure. Both are longitudinal

(ii) A water wave of wavelength 50m travels at 12m/s. How many waves will pass in a minute?

$c = \frac{\lambda}{T}$ $\therefore T = \frac{\lambda}{c} = \frac{50\text{m}}{12\text{m/s}} = 4.17\text{s}$

period

So # waves per minute is $60/T$

$= \boxed{14.4}$ (So 14 full waves)

(iii) UVA rays have a wavelength of about 350nm, whereas UVC has a wavelength of about 200nm. UVC is absorbed by the atmosphere whereas UVA ~~are~~ is not, which is why you should apply sunscreen when going out in strong sunlight.

Calculate the *frequencies* (in Hz) and *periods* (in s) for UVA and UVC, using standard form.

$f = \frac{c}{\lambda}$

$T = \frac{1}{f} = \frac{\lambda}{c}$

$f_{UVA} = \frac{2.998\times10^{8}\text{m/s}}{350\times10^{-9}\text{m}} = \boxed{8.57\times10^{14}\text{ Hz}}$

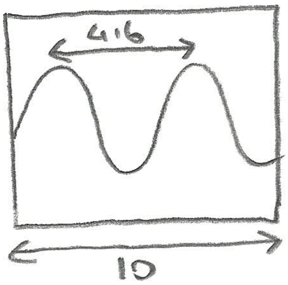
$T_{UVA} = \boxed{1.17\times10^{-15}\text{s}}$

$f_{UVC} = \frac{2.998\times10^{8}\text{m/s}}{200\times10^{-9}\text{m}} = \boxed{1.50\times10^{15}\text{ Hz}}$

$T_{UVC} = \boxed{6.67\times10^{-16}\text{s}}$

Question set D

(i) An oscilloscope is used to determine the period and hence the frequency of a wave. On the screen of the oscilloscope the difference between peaks of an oscillation is 4.6 squares. Calculate (a) the period /ms and (b) the frequency /kHz if the timebase is set at 0.5ms.



$T = 4.6 \times 0.5 \text{ ms}$

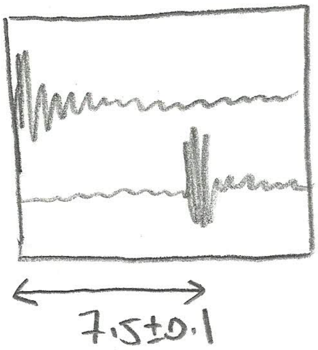
$= \boxed{2.3\text{ms}}$

$\therefore f = \frac{1}{T} = \frac{1}{2.3 \times 10^{-3}\text{s}} = 434.8 \text{ Hz}$

$= \boxed{0.43 \text{ kHz}}$

(i) A two-input storage scope is used to determine the time delay between waves arriving at two microphones displaced by a distance of 5.12m.

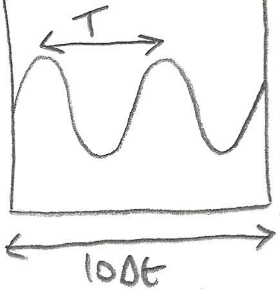
The number of oscilloscope squares that correspond to the time delay between the signals is 7.5 ± 0.1. If the timebase is set at 2ms, calculate an *inequality* which bounds a measurement of the speed of sound /ms^{-1}. Assume the distance between the microphones is perfectly precise.



$\frac{5.12}{7.6 \times 2 \times 10^{-3}} < c/\text{ms}^{-1} < \frac{5.12}{7.4 \times 2 \times 10^{-3}}$

$\therefore \boxed{337 \text{ ms}^{-1} < c < 346 \text{ ms}^{-1}}$

(iii) BBC Radio 4 is transmitted on a carrier radio wave at a frequency of about 99MHz. A special oscilloscope is needed to measure the period of this wave. If *two complete waves* can fit into the oscilloscope screen (which has ten divisions horizontally), work out the timebase needed.



$\uparrow \Delta t$

So $2T > 10\Delta t$

$\frac{T}{5} > \Delta t$

to fit at least two waves

$T = \frac{1}{f}$ So $\boxed{\Delta t < \frac{1}{5f}}$

$\therefore \Delta t < \frac{1}{5 \times 99 \times 10^{6}} \text{ s}$

$\boxed{\Delta t < 2.0 \times 10^{-9} \text{ s}}$

(So about 2 ns timebase needed)

What is the wavelength of the Radio 4 carrier wave?

$c = f\lambda \therefore \lambda = \frac{c}{f}$

$\lambda = \frac{2.998 \times 10^{8} \text{ m/s}}{99 \times 10^{6} \text{ Hz}} = \boxed{3.03 \text{ m}}$

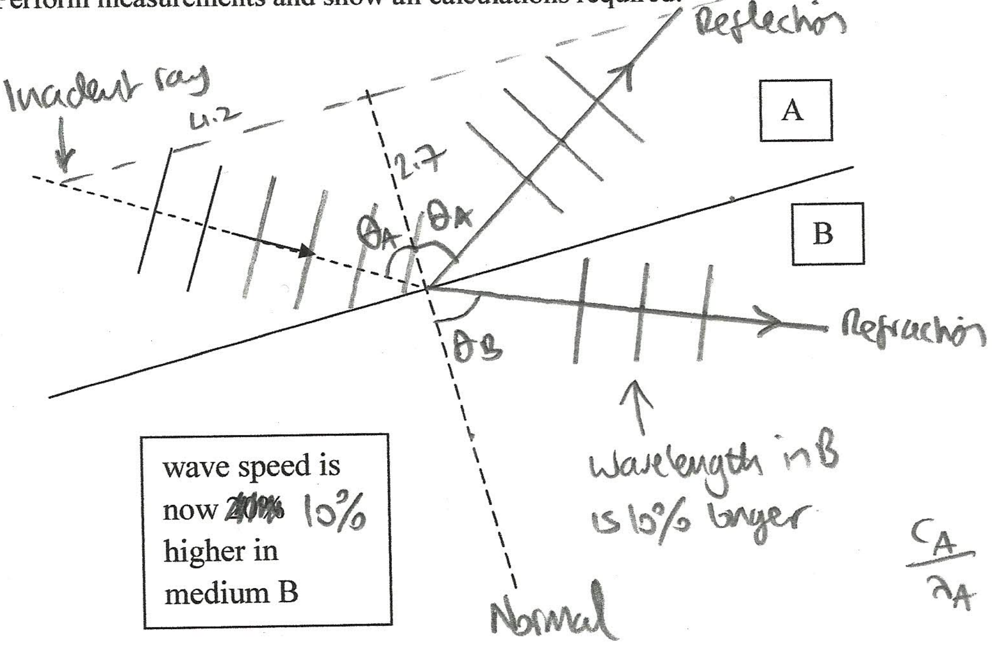
Question set E

(i) You will need (a) a sharp pencil (b) a ruler (c) a protractor to answer this question. Obtain this kit before you proceed!

For each scenario, mark additional wavefronts to show how a wave *both* **reflects** and **refracts**. Perform measurements and show all calculations required.

$\theta_A = \tan^{-1}\left(\frac{4.2}{2.7}\right)$

$= \boxed{57.3^\circ}$



$\boxed{\text{Snell's law}}$ $c_B = 1.1 c_A$

$\frac{\sin\theta_A}{c_A} = \frac{\sin\theta_B}{1.1 c_A}$

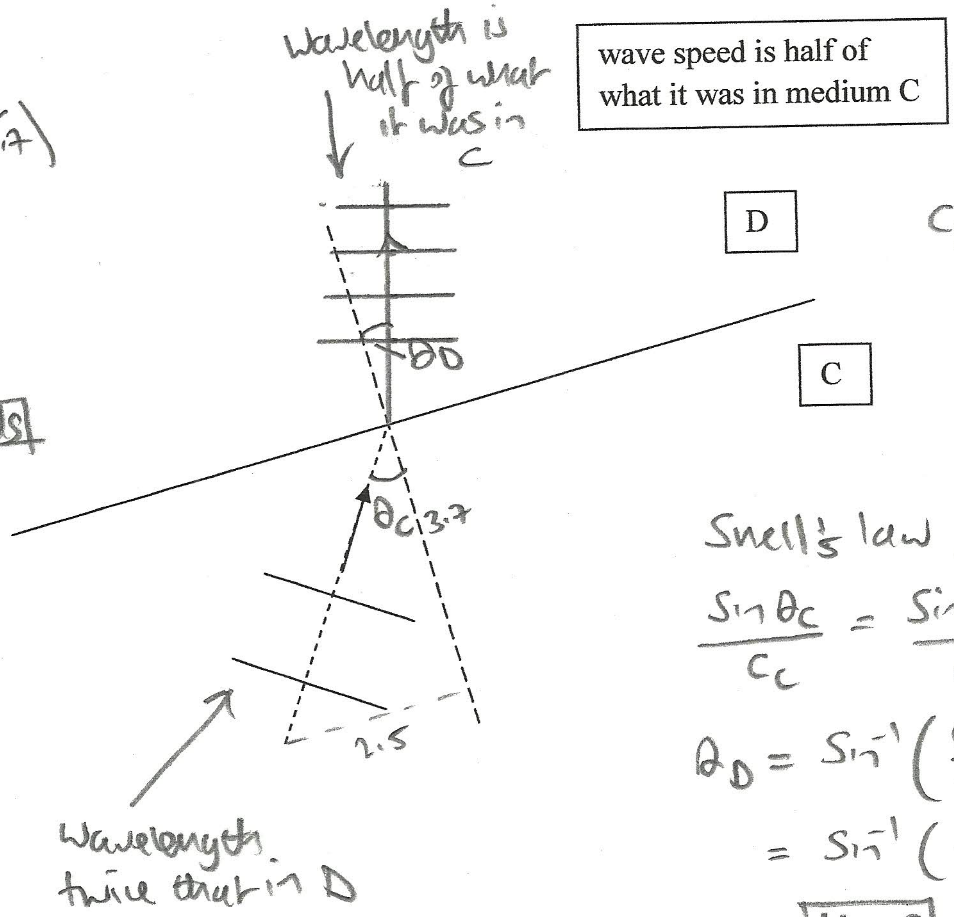
$\therefore \theta_B = \sin^{-1}(\sin\theta_A \times 1.1)$

$= \boxed{67.7^\circ}$

$\frac{c_A}{\lambda_A} = \frac{c_B}{\lambda_B}$ so $\boxed{\lambda_B = 1.1\lambda_A}$

$\theta_C = \tan^{-1}\left(\frac{2.5}{3.7}\right)$

$= \boxed{34.0^\circ}$



$c_D = \frac{1}{2} c_C$

Waves are refracted $\boxed{\text{towards}}$ $\boxed{\text{the normal}}$ if wave speed slows down.
(for light ⇒ higher refractive index)

Snell's law of refraction

$\frac{\sin\theta_C}{c_C} = \frac{\sin\theta_D}{c_D}$

$\theta_D = \sin^{-1}\left(\frac{c_D}{c_C}\sin\theta_C\right)$

$= \sin^{-1}\left(\frac{1}{2}\sin\theta_C\right)$

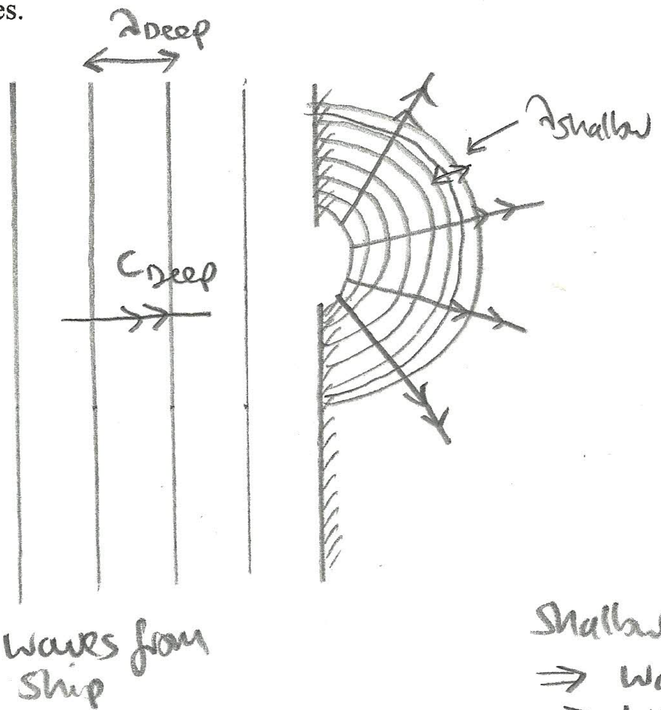
$= \boxed{16.12^\circ}$

(ii) A water wave from a passing cruise ship enters a shallow harbour. The wavelength of the wave when it enters the harbour is about the same width as the harbour entrance, which is narrow compared to the harbour itself. The harbour is significantly less deep than the water at the entrance to the harbour.

Draw a plan view to describe the wave pattern that you would expect to see at the wave enters the harbour. Clearly explain and annotate the key features.

f constant so

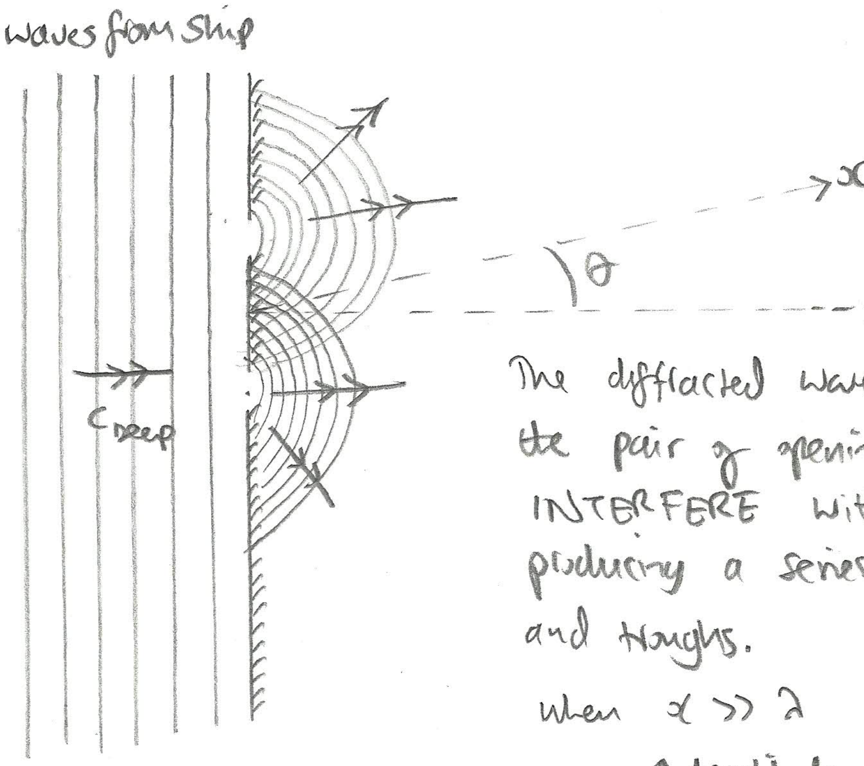
$$\frac{c_{deep}}{\lambda_{deep}} = \frac{c_{shallow}}{\lambda_{shallow}}$$



Shorter wavelength waves DIFFRACT from harbour entrance

Shallow harbour
⇒ wave speed less
⇒ wavelength less

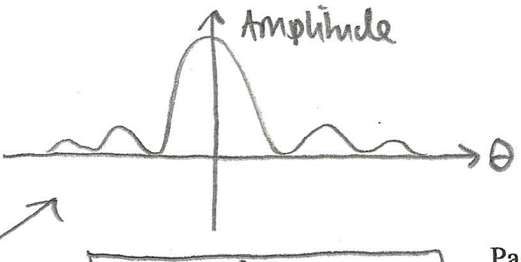
(ii) **Challenge** The same waves enter a nearby harbour, but this one has a separate entrance and exit. These are similar widths as above, and they are separated by about four widths. Sketch the wave pattern that you would expect to see at the wave enters the harbour. Clearly explain and annotate the key features.



[Max along this line]

The diffracted waves from the pair of openings INTERFERE with each other producing a series of peaks and troughs.

when $x \gg \lambda$



This is called a $\boxed{\text{diffraction pattern}}$