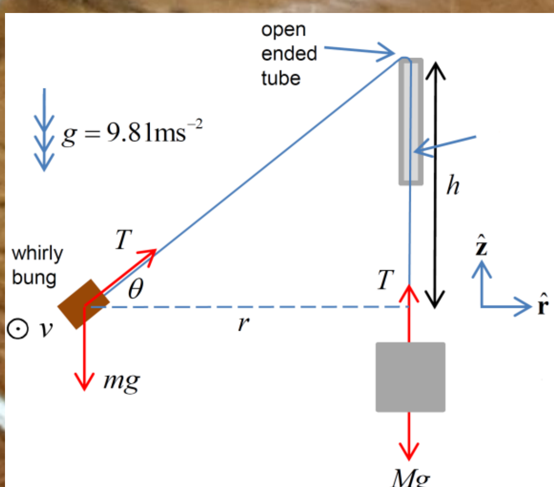
TEACHER NOTES

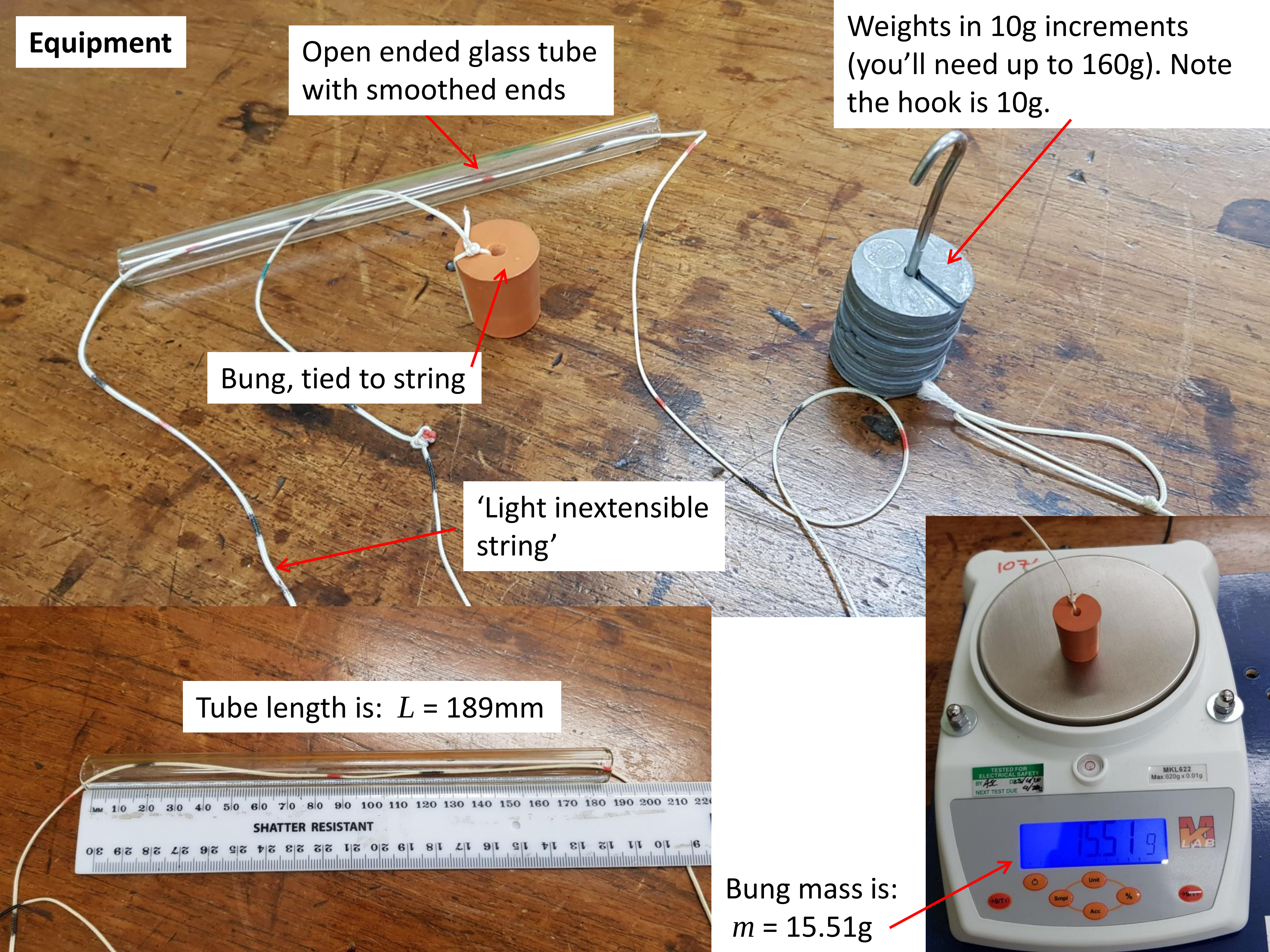
$$\sin\theta = \frac{m}{M}$$

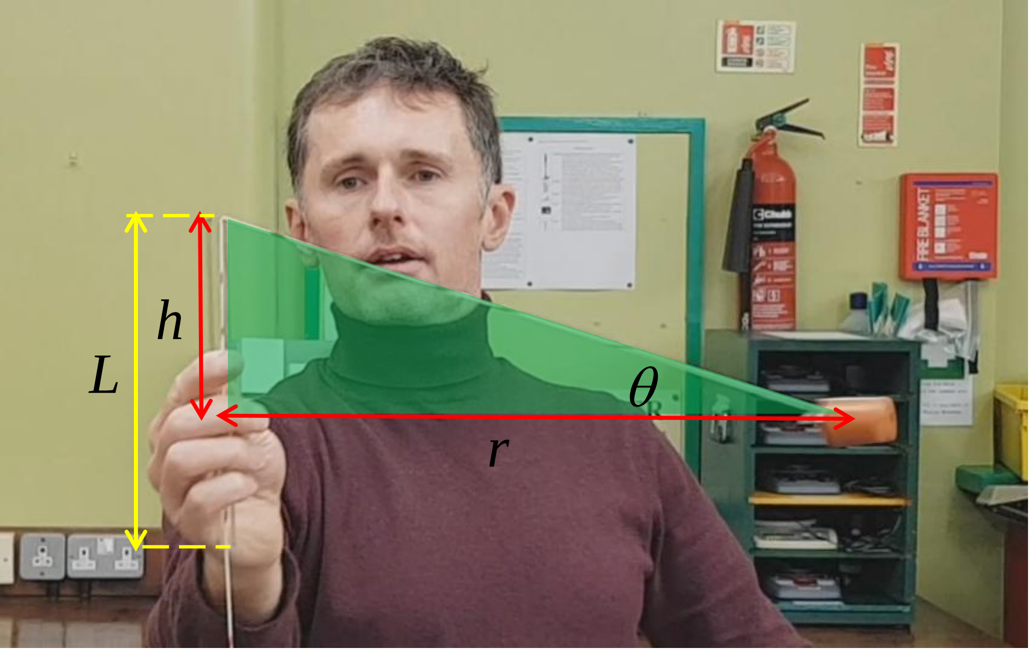
$$r = \frac{gP^2}{4\pi^2 \tan\theta}$$

Whirling bungs



Dr Andrew French. October 2020.


Equipment
Open ended glass tube with smoothed ends
Weights in 10g increments (you'll need up to 160g). Note the hook is 10g.
Bung, tied to string
'Light inextensible string'
Tube length is: L = 189mm
SHATTER RESISTANT
Bung mass is: m = 15.51g



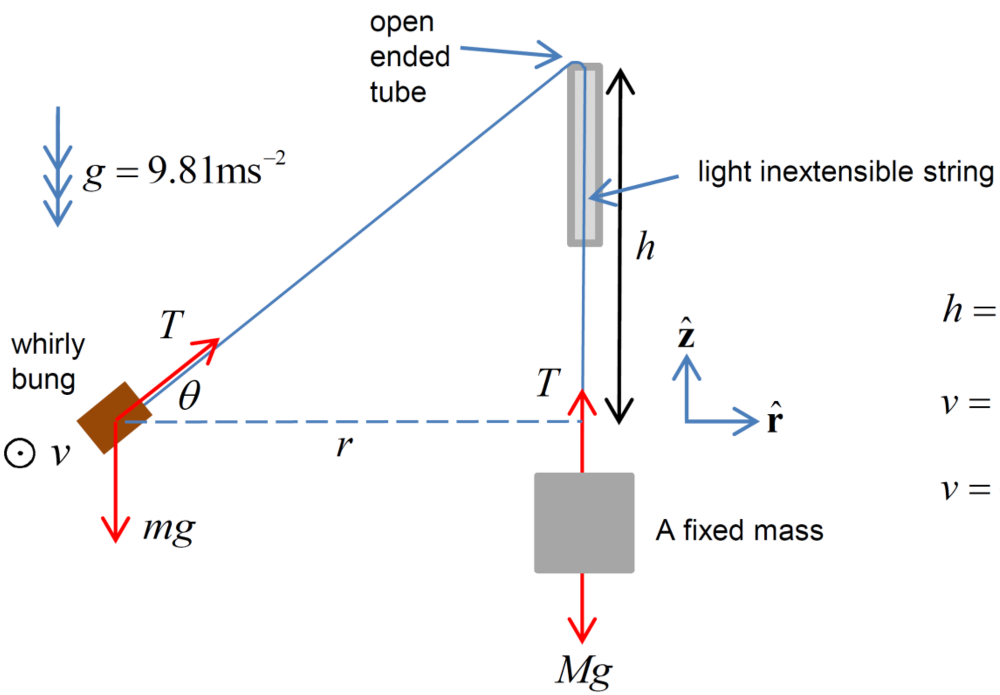
Hook masses in range 40g to 160g into a loop of the string, which passes through the glass tube and is tied to the bung. Dangle masses vertically.

Whirl the bung until the strings stops moving inside the tube.

Get a partner to video about ten rotations using a digital camera or smartphone.

Copy the video files to a laptop, and name each one according to the added mass.

Take a freeze-frame for each movie and use this to calculate L, h, r and angle θ (from pixels, or direct measurement from a screen).

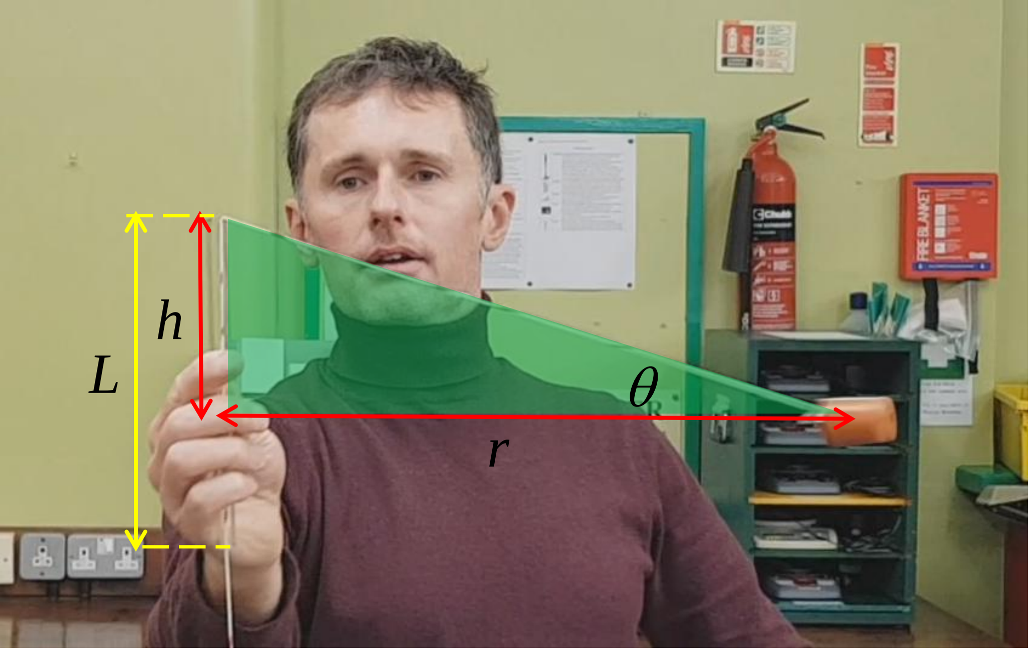


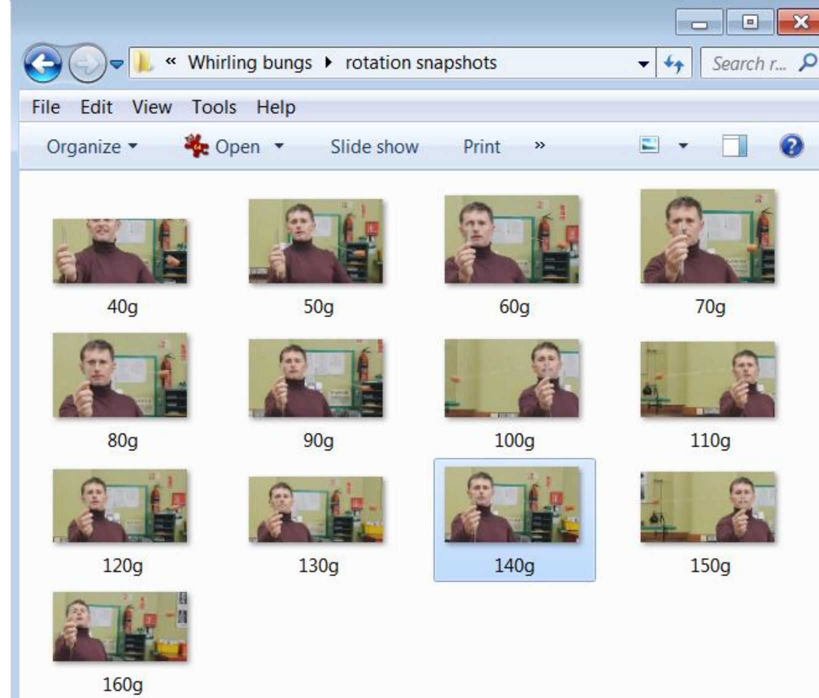
$$h = r\tan\theta$$

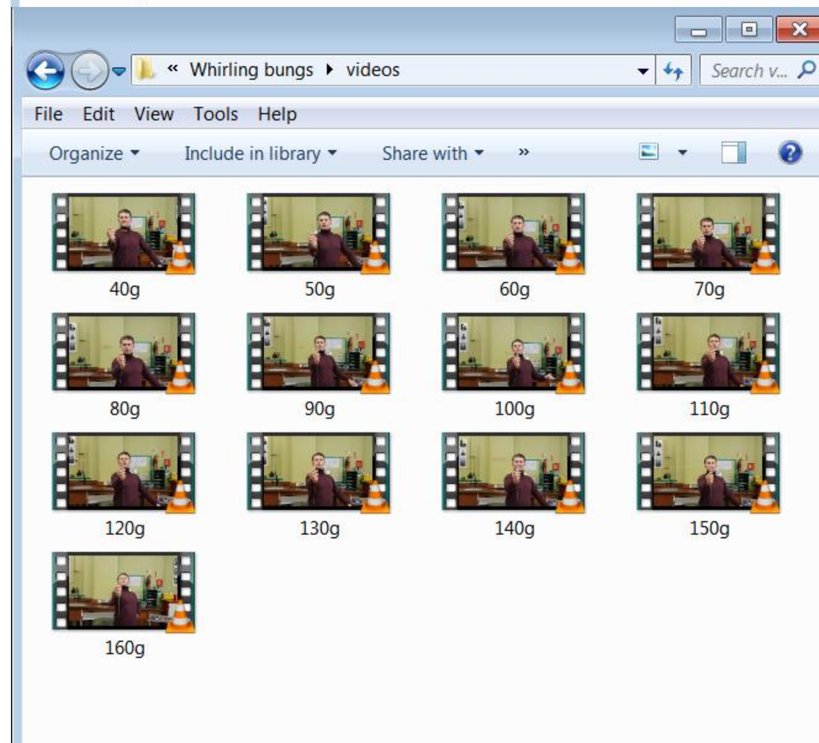
$$v = r\omega$$

$$v = \frac{2\pi r}{P}$$

Time for one rotation i.e. 'period'





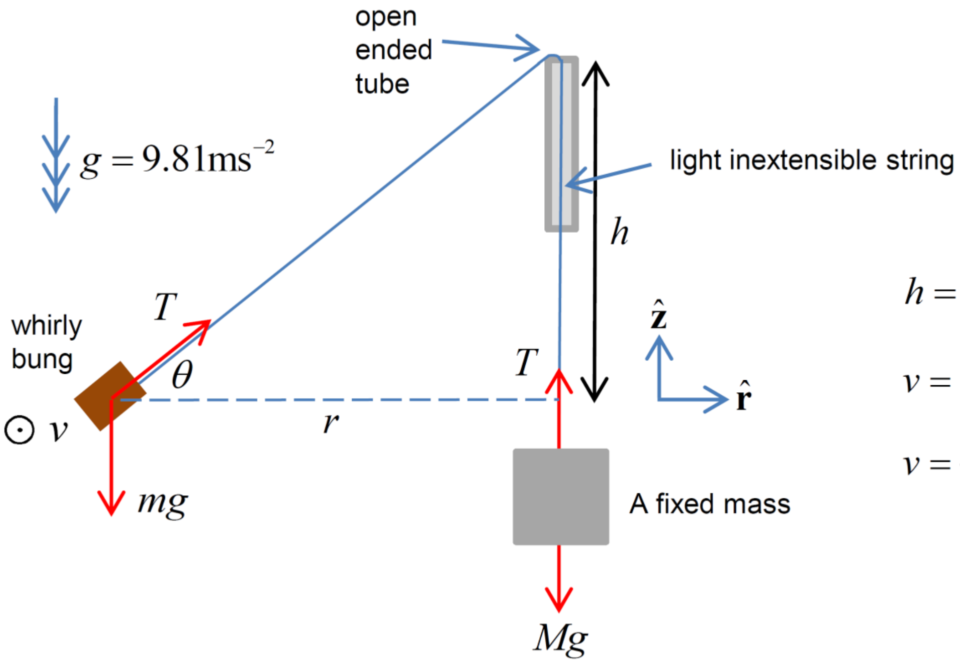


Take a screenshot of a freeze-frame from each movie and copy and paste into a bitmap editor such as IrfanView. Use the **crop tool** to determine the **pixel values** of L, h, r.

Since we know the actual length of the tube in mm, we can convert these pixel values into measurements in mm.

$$\theta = \tan^{-1}\left(\frac{h}{r}\right) \qquad r = \frac{r}{L} \times 189\text{mm}$$

Screen pixel to reality *calibration*



Model predictions

$$\sin\theta = \frac{m}{M}$$

$$r = \frac{gP^2}{4\pi^2 \tan\theta}$$

$$h = r\tan\theta$$

$$v = r\omega$$

$$v = \frac{2\pi r}{P}$$

Time for one rotation i.e. 'period'

Newton II for bung

$$//\ \hat{\mathbf{r}}: \quad \frac{mv^2}{r} = T\cos\theta$$

$$//\ \hat{\mathbf{z}}: \quad 0 = T\sin\theta - mg$$

Newton II for mass

$$//\ \hat{\mathbf{z}}: \quad 0 = T - Mg$$

Hence:

$$T\sin\theta = mg$$

$$T = Mg$$

$$\therefore \sin\theta = \frac{m}{M}$$

$$\therefore \boxed{\theta = \sin^{-1}\left(\tfrac{m}{M}\right)}$$

$$T\cos\theta = \frac{mv^2}{r}$$

$$T\sin\theta = mg$$

$$\therefore \tan\theta = \frac{gr}{v^2} = \frac{grP^2}{4\pi^2 r^2}$$

$$\therefore P = 2\pi\sqrt{\frac{r\tan\theta}{g}}$$

$$\therefore \boxed{P = 2\pi\sqrt{\frac{h}{g}}}$$

To measure r, mark string in cm spacings and hence measure bung to tube length

$$l = \sqrt{r^2 + h^2}$$

$$\therefore \boxed{r = \sqrt{l^2 - h^2}}$$

Use **video capture** to measure h, *calibrated* by direct measurement of l

Analysis

The natural *variables* for the whirly bungs experiment are the mass of the fixed mass M, and the period of rotation P.

Time ten rotations and then average

Predicted quantities are the angle θ and the radius r

For a typical bung $m = 14\text{g}$

$$\sin\theta = \frac{m}{M}$$

$$r = \frac{gP^2}{4\pi^2 \tan\theta}$$

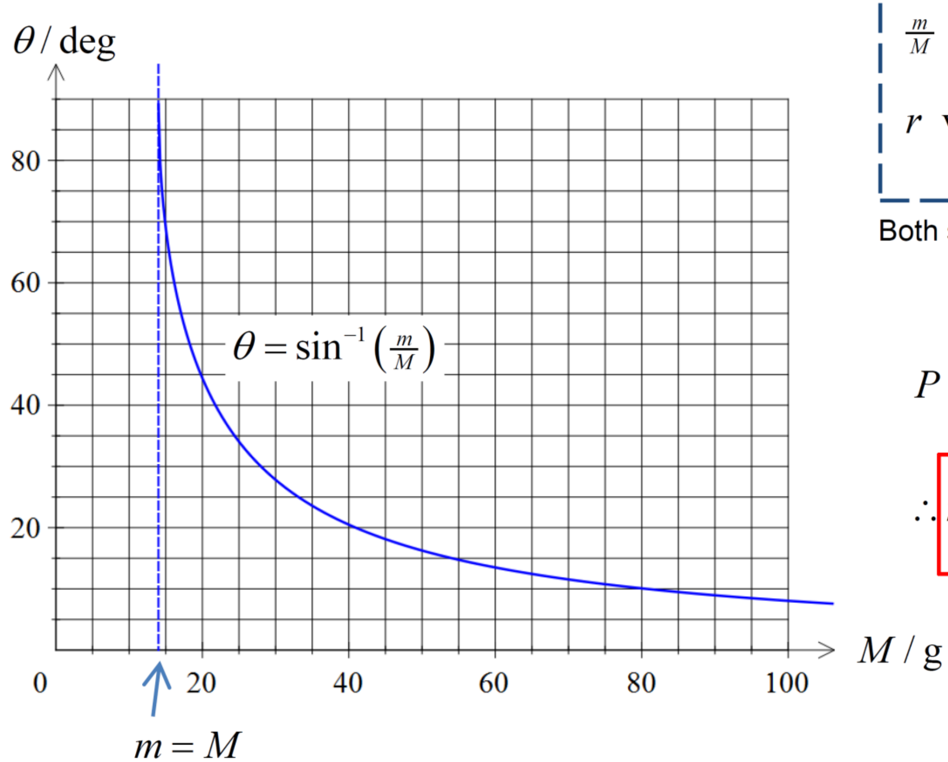
For **analysis** plot:

$\frac{m}{M}$ vs $\sin\theta$

r vs $\frac{P^2}{\tan\theta}$

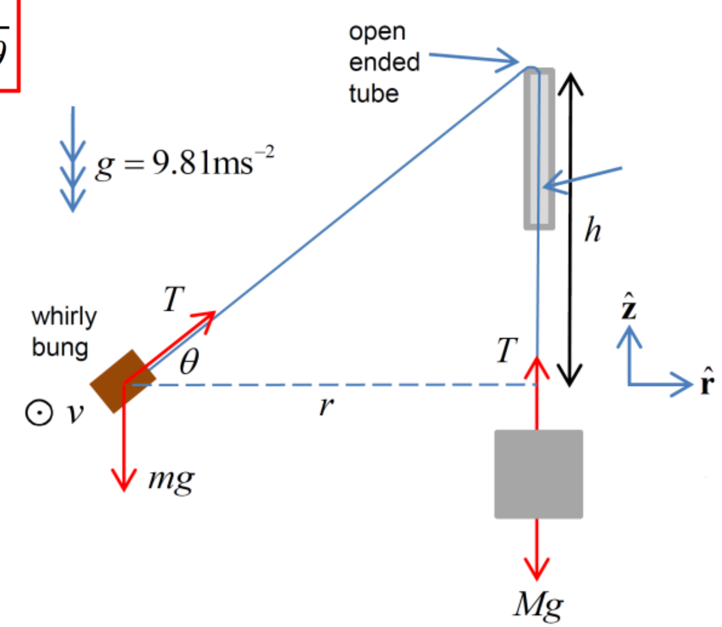
Both should be straight lines

i.e. plot model vs prediction. Should be a $y = x$ line if perfectly correlated.



$$P = 2\pi\sqrt{\frac{r\tan\theta}{g}}$$

$$\therefore \boxed{r = \frac{gP^2}{4\pi^2\tan\theta}}$$



WHIRLING BUNGS

Andy French. Winchester College P5. 30/10/2020.

Measurements from video motion capture.

Period timings from replay of videos.

Radius and angle measurements from screenshot of videos.

Bung mass m /g	15.51
Tube length /mm	189
Strength of gravity /Nkg^-1	9.81

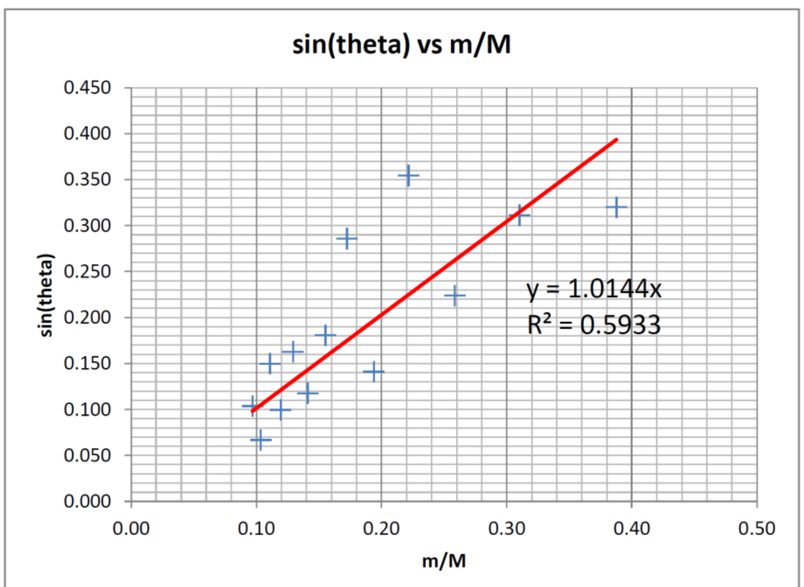
Compared to most other A-Level experiments, it is difficult to obtain a strong correlation of model and measurement. The **sin(theta) = m/M** model is a reasonable fit (it has the correct 1:1 gradient!), but overall the degree of correlation is poor. The plot of measured vs model radius is *entirely* inconclusive.

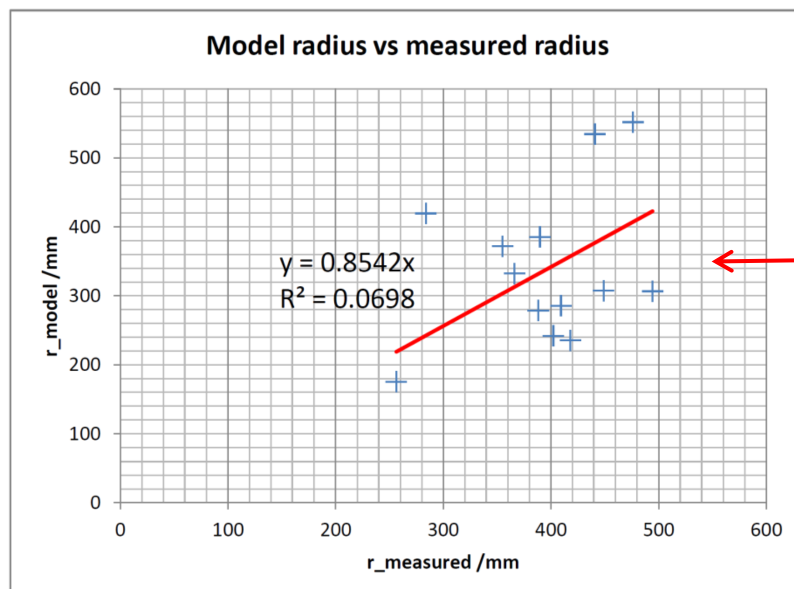
Many potential sources of error:

- Significant friction between string and glass lip, and expect this to increase as bung is whirled faster.
- Glass lip is often not flat, resulting in increased elevation.
- The 29FPS frame rate results in blurry images, so precise measurement from screenshots is difficult.

In Quicktime use frame number instead of time /s. FPS = 29.

Mass M /g	Radius r /pix	height /pix	Tube length /pix	r /mm	theta /rad	theta /deg	t1 /s	t2 /s	# periods	Period P /s	m/M	sin(theta)	r = (g*P^2)/(4*pi^2 * tan(theta)) /mm
40	595	201	269	418	0.326	18.7	1.24	4.07	5	0.57	0.39	0.320	235
50	525	172	271	366	0.317	18.1	1.24	4.55	5	0.66	0.31	0.311	332
60	479	110	255	355	0.226	12.9	0.66	7.69	12	0.59	0.26	0.224	372
70	343	130	253	256	0.362	20.8	0.41	5.07	9	0.52	0.22	0.354	175
80	407	58	271	284	0.142	8.1	0.76	5.17	9	0.49	0.19	0.141	419
90	466	139	219	402	0.290	16.6	2.38	6.69	8	0.54	0.17	0.286	242
100	495	91	241	388	0.182	10.4	1.24	3.97	6	0.45	0.16	0.181	279
110	532	63	258	390	0.118	6.8	0.79	3.79	7	0.43	0.14	0.118	385
120	576	95	266	409	0.163	9.4	0.69	4.17	8	0.44	0.13	0.163	286
130	670	67	266	476	0.100	5.7	1.07	5.31	9	0.47	0.12	0.100	552
140	627	95	264	449	0.150	8.6	0.93	4.83	9	0.43	0.11	0.150	307
150	583	39	250	441	0.067	3.8	0.69	3.72	8	0.38	0.10	0.067	534
160	633	66	242	494	0.104	6.0	0.10	1.90	5	0.36	0.10	0.104	307



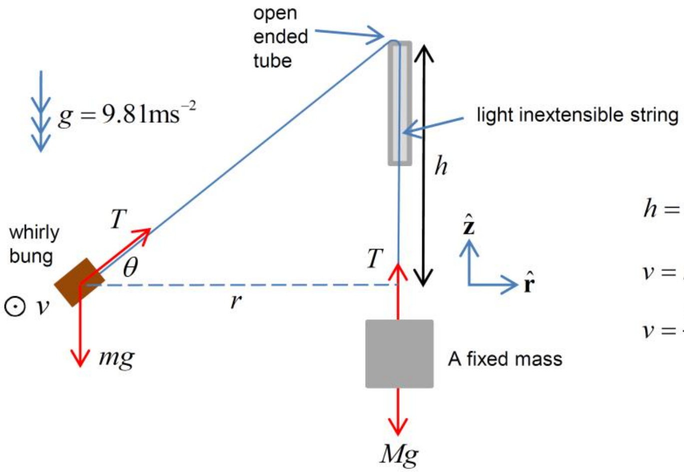


In both cases, prediction is plotted against measurement.

If 100% correct this should be the line $y = x$.

Whirly Bungs experiment

This simple experiment is an example of horizontal circular motion. A rubber bung, attached to a fixed mass via a light inextensible string threaded through an open ended tube, is whirled in a horizontal circle until 'dynamic equilibrium' is obtained. i.e. the radius of circular motion is *constant*, as is the angle of inclination.



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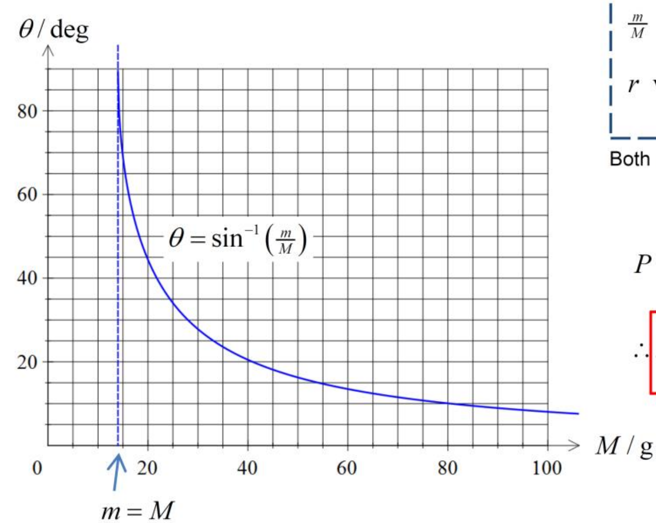

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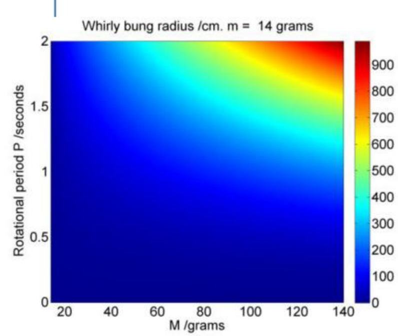

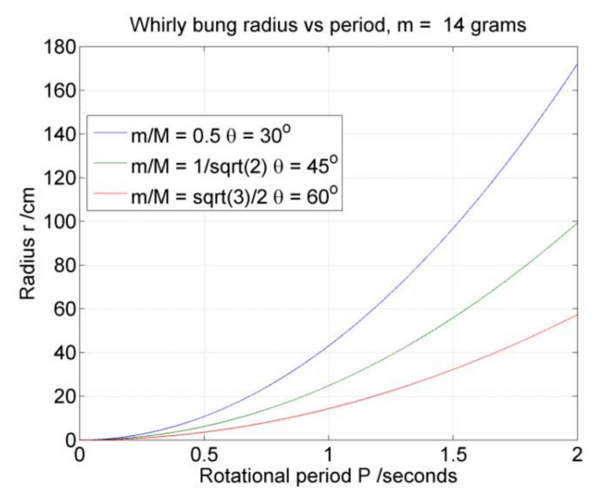

From Eclecticon Circular Motion note