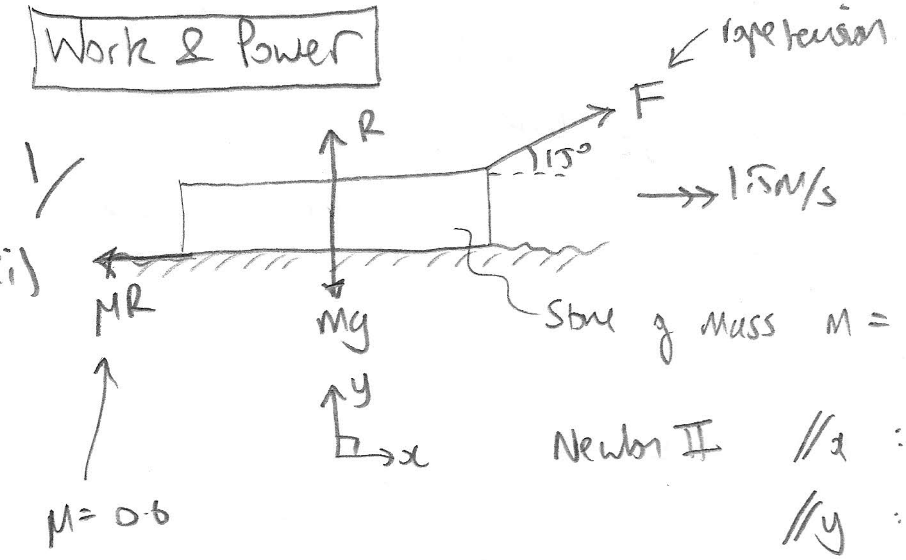
Work & Power

1/

(i)



$g = 9.81$ N/kg

Newton II $/\!/x$: $0 = F\cos 15° - \mu R$ ①

$/\!/y$: $0 = F\sin 15° + R - Mg$ ②

Dynamic equilibrium since moving at constant speed.

Work done per second is $\boxed{F\cos 15° \times 1.5\,\text{m/s}}$, so need to find F.

①: $R = \dfrac{F\cos 15°}{\mu}$ ∴ in ②: $F\sin 15° + \dfrac{F\cos 15°}{\mu} = Mg$

$\therefore F\left(\sin 15° + \dfrac{\cos 15°}{\mu}\right) = Mg$

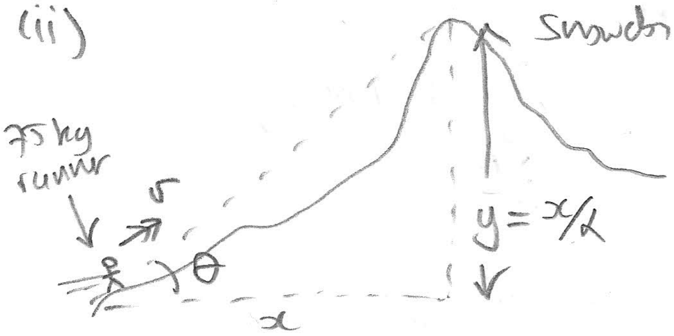
$\therefore F\cos 15° \times 1.5 = \dfrac{Mg\cos 15° \times 1.5}{\sin 15° + \dfrac{\cos 15°}{0.6}}$

$= \dfrac{25\times10^3 \times 9.81 \times \cos 15° \times 1.5}{\sin 15° + \cos 15°/0.6}$

$= \boxed{190\text{ kW}}$

$(1.90\times10^5\text{ J/s})$.

(ii)



$g = 9.81\,N/kg$

$y = 965\,M$

(Not too scale!)

gradient $\alpha = 7.1$ (ie "1 in 7.1")

Rate of work done against gravity is $\boxed{P = \frac{mgy}{t}}$

$= \frac{75\,kg \times 9.81\,N/kg \times 965\,M}{40 \times 60\,s} = \boxed{296\,W}$

Distance travelled is $\sqrt{x^2 + \frac{x^2}{\alpha^2}}$, so speed // slope is:

y^2

$v = \frac{x\sqrt{1 + 1/\alpha^2}}{t}$ and $x = 7.1 \times 965\,M$

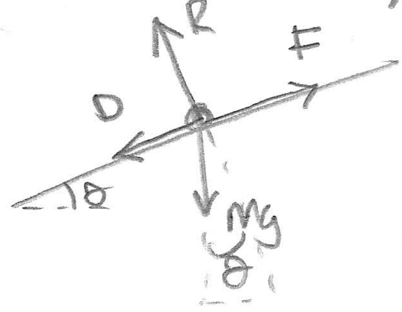
ie $v = \frac{7.1 \times 965\sqrt{1 + 1/7.1^2}\,m}{40 \times 60\,s} = \boxed{2.88\,M/s}$

If driving force // slope is F, Fv = rate of work done against gravity if drag and friction can be ignored.

$\therefore F = 296\,W / 2.88\,M/s = \boxed{103\,N}$

[Note since in eg we can also compute the net resistive force D // slope.

$\rightarrow 2.88\,M/s$ ← ie not accelerating.



● = runner (!)

NII // to slope:

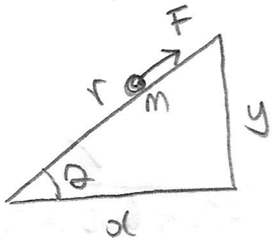
$F = D + mg\sin\theta$

$\therefore D = 103 - 75 \times 9.81 \times \sin(\tan^{-1}(\frac{1}{7.1}))$

$\simeq 0\,N$]

This is exactly what we would expect if $F \times$ slope distance equates to the GPE change. If there are additional resistive forces, then $F \times$ slope distance must account for work done against these forces as well as gravity.

Additional note: a slope provides a <u>mechanical advantage</u>



eg let $Fr = Mgy$ ie work done pushing particle of mass M up a slope $=$ GPE change.

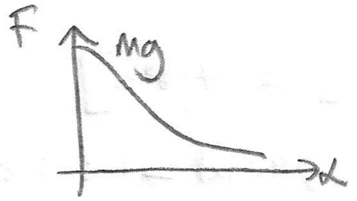
Now $r = \sqrt{x^2+y^2}$ and let $x = \alpha y$ where gradient is "1 in α".

$$\therefore \quad r = y\sqrt{1+\alpha^2}$$

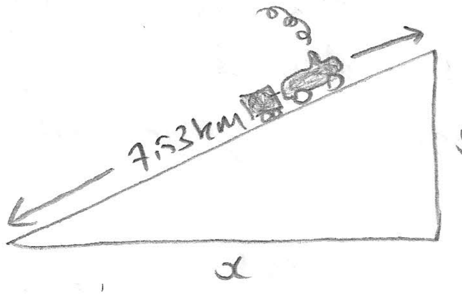
$$\therefore \quad Fy\sqrt{1+\alpha^2} = Mgy$$

$$\therefore \quad \boxed{F = \frac{Mg}{\sqrt{1+\alpha^2}}}$$

So $\sqrt{1+\alpha^2}$ is the mechanical advantage, ie what we can divide the weight by to get the pushing force F. A <u>vertical</u> slope means $\alpha \to 0 \Rightarrow F = Mg$ ie no mechanical advantage at all.



(iii)



$\cancel{\#} g = 9.81\ N/kg$

$x = \alpha y$ $\quad \boxed{\alpha = 7.86}$

1h 45 minutes

Engine power is $\boxed{P = 238\ kW}$

Journey time (at constant speed) is $\boxed{45 \times 60 s = t}$

If engine power = rate of work done against gravity

$P = mgy/t$ (if ignore resistive forces // track).

$\therefore \quad M = Pt/gy$ where M is the total mass of the train + carriage + passengers.

It is a maximum since if any resistive forces, some of P will be expended doing work against these, not just gravity.

$r^2 = x^2 + y^2$ $\quad r = 7530\,M$

$r^2 = \alpha^2 y^2 + y^2$

$r = \sqrt{\alpha^2+1}\ y$ $\quad \therefore \quad y = \dfrac{r}{\sqrt{\alpha^2+1}}$

$$\therefore \quad \boxed{M = \frac{Pt}{gr}\sqrt{\alpha^2+1}}$$

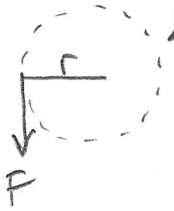
[$\alpha = 7.86$]

gradient = 1 : 7.86

$$\therefore \quad M = \frac{238 \times 10^3 \times 45 \times 60}{9.81 \times 7530}\sqrt{7.86^2+1}$$

$= \boxed{6.89 \times 10^4\ kg}$ (68.9 tonnes)

(iv)



path of pedals on bike.

Assume by using alternate pedals F is a constant.

Work done / rotation is $F \times 2\pi r$

If f is rotations per minute then time per rotation is $T = \frac{1}{f/60} = \frac{60}{f}$ seconds

$\therefore$ rate of work done is $P = \frac{F \times 2\pi r}{60/f}$

$$P = \frac{2\pi}{60} Frf$$

$$\boxed{P = \frac{\pi}{30} Frf}$$

So if $P = 180W$, $r = 0.17m$, $F = 200N$

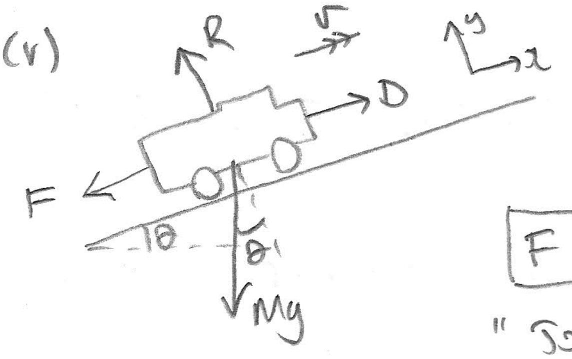
$$\Rightarrow \boxed{f = \frac{30P}{\pi F r}}$$

$$f = \frac{30 \times 180}{\pi \times 200 \times 0.17} = \boxed{50.6}$$ rotations per minute (RPM)

[Note moving to a $\boxed{\text{lower gear}}$ will mean less force is required to rotate the pedals. However, to maintain the same speed you will need to still supply the same power. Hence the trade off for lower F is a proportional increase in rotation rate f].

ie, assume balance of drive force P/v and drag + resistive forces.

(v)



$\theta = 3^\circ$.

$g = 9.81\,N/kg$

$\boxed{F = kMg\sin\theta}$ $k = 0.5$

"50% of component of weight acting down the slope = drag"

if at constant speed $v = \frac{50 \times 10^3}{3600}$ m/s $= \boxed{13.9\ m/s}$

and no acceleration $\therefore$ $D = F + Mg\sin\theta$

$\therefore$ $\boxed{D = (1+k)Mg\sin\theta}$ Driving force

let engine power P be such that $\boxed{\eta P = Dv}$

η = efficiency (in our case $\eta = 0.25$).

"force × velocity"

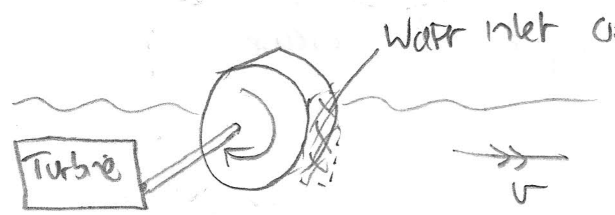
$\therefore$ $\boxed{P = \frac{(1+k)Mg\sin\theta\, v}{\eta}}$

$\therefore$ $P = \frac{(1+0.5) \times 10^4 \times 9.81 \times \sin 3^\circ \times \frac{50\times10^3}{3600}}{0.25}$

(10 tonne $= 10^4$ kg)

$\boxed{P = 428\ kW}$

(vi)



Stream. $\rho = 1000\,kg/m^3$

output power $= P = 27$ kW

Efficiency $\eta = 0.2$.

let $\boxed{\eta \times \frac{1}{2}mv^2 = P}$

where m is the mass of water flowing through the wheel per second

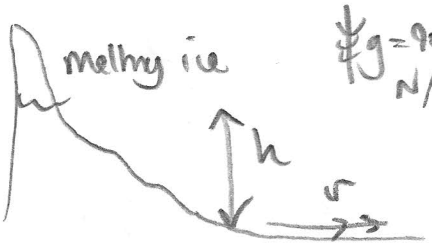
(i.e. power $= 0.2 \times$ KE transformed per second)

If stream moving at v m/s enters inlet A

$\Rightarrow$ $\boxed{\dot{m} = \rho A v}$

So $P = \eta \times \frac{1}{2}\rho A v^3$ $\therefore$ $\boxed{v = \sqrt[3]{\dfrac{2P}{\eta \rho A}}}$

$\therefore v = \sqrt[3]{\dfrac{2 \times 27 \times 10^3}{0.2 \times 1000 \times 0.5}} = \boxed{8.14 \text{ m/s}}$



mountain: source of water

$g = 9.81$ N/kg

If $mgh = \frac{1}{2} m v^2$

i.e. GPE $\rightarrow$ water KE (with no frictional losses)

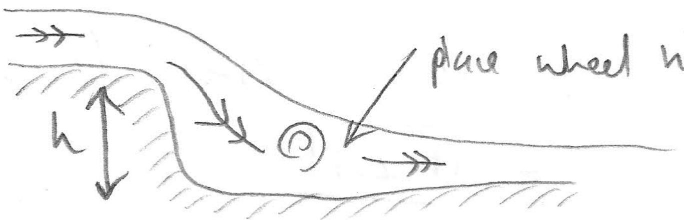
$\Rightarrow$ $\boxed{h = \frac{1}{2} v^2 / g}$

Actually don't need much of a drop height!

$\therefore h = \frac{1}{2} \times 8.14^2 / 9.81$

$\boxed{h = 3.38 \text{ m}}$

So more like:



Note: if $v^2 = 2gh$ $\therefore$ $v = \sqrt{2gh}$

$\therefore P = \eta \times \frac{1}{2} \rho A (2gh)^{3/2}$

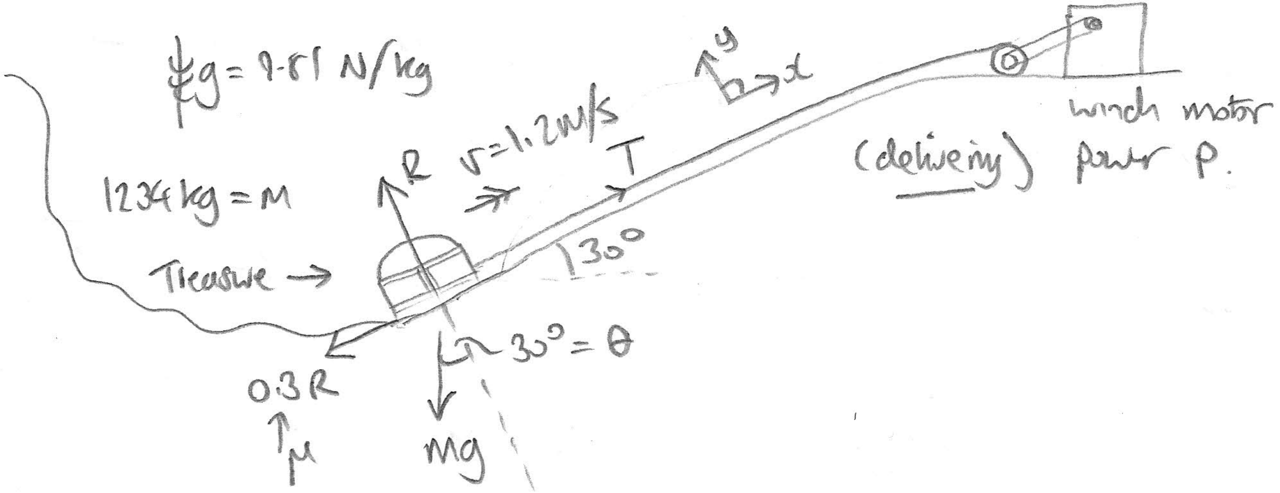
So $\boxed{\dfrac{1}{2g}\left(\dfrac{2P}{\eta \rho A}\right)^{2/3} = h}$

1.8 GW !!

{ Dinorwig in Wales
"Electric Mountain"
$h \simeq 900$ m, $P = 1800$ MW
$\eta \simeq 0.2$; $\rho = 1000$ kg/m^3
$\Rightarrow$ $\boxed{A = 7.67 \text{ m}^2}$ }

i.e. a pipe of ≈ 1.6 m radius. [$\pi r^2 = A$]

(vii) $g = 9.81\ N/kg$



Newton II $//x$: $\quad 0 = T - \mu R - mg\sin\theta \quad ①$

$//y$: $\quad 0 = R - mg\cos\theta \quad ②$

Since moving at 1.2 m/s and ∴ in eq.

$\therefore \quad T = \mu R + mg\sin\theta \qquad R = mg\cos\theta$

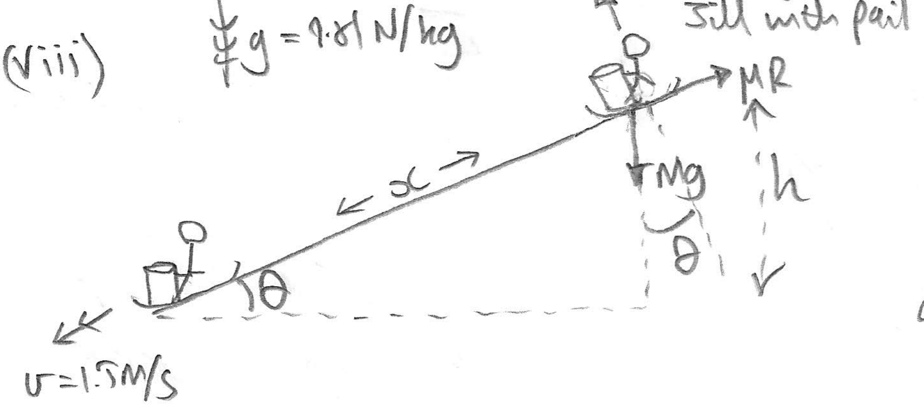
$\therefore \boxed{T = mg(\mu\cos\theta + \sin\theta)}$

Now $\quad P = Tv \qquad \therefore \boxed{P = mgv(\mu\cos\theta + \sin\theta)}$

$\therefore P = 1234 \times 9.81 \times 1.2\,(0.3\cos 30° + \sin 30°)$

$= \boxed{11\ kW}$

(viii) $g = 9.81\ N/kg$



Conservation of energy

$$\underbrace{mgh}_{\text{GPE at top of hill}} = \underbrace{\tfrac{1}{2}mv^2}_{\text{KE at bottom of hill}} + \underbrace{\mu R x}_{\text{work done against friction}}$$

$\tan\theta = \dfrac{1}{\alpha}$

so $\alpha = \dfrac{1}{\tan\theta} = \cot\theta$

Geometry: $h = x\sin\theta$

NII $\perp$ slope: $R = Mg\cos\theta$

$\therefore \quad mgx\sin\theta = \frac{1}{2}mv^2 + \mu x\, mg\cos\theta$

$\sin\theta = \frac{1}{2}\frac{v^2}{gx} + \mu\cos\theta$

$\therefore \quad \sin\theta - \mu\cos\theta = \frac{v^2}{2gx}$ } Can't easily solve for θ without a numeric method

↳ so change question to "Find μ given θ"

$$\therefore \quad \boxed{\mu = \frac{\sin\theta - \frac{v^2}{2gx}}{\cos\theta}}$$

let $\theta = 10^\circ$

$$\therefore \quad \mu = \frac{\sin 10^\circ - \frac{1.5^2}{2\times9.81\times20}}{\cos 10^\circ} = \boxed{0.171}$$

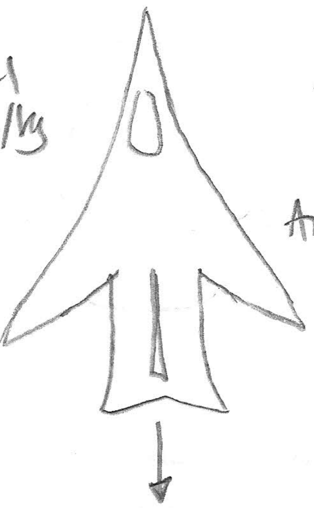
[Note if $v = 0$, ie Jill is balanced on the slope and not moving, $\mu = \tan\theta = 0.176$]

If Jill slides at 3 m/s: $\mu = \frac{\sin 10^\circ - \frac{3^2}{2\times9.81\times20}}{\cos 10^\circ}$

$= \boxed{0.153}$

(ix) (a) $\uparrow 2 \times P/v$ (Two engines, power P each)

$g = 9.81$ N/kg



$\uparrow v = 250$ m/s

Aircraft, mass m

$mg + \frac{1}{2}C_D\rho A v^2$

weight drag

Newton II:

$$0 = \frac{2P}{v} - mg - \frac{1}{2}C_D\rho A v^2$$

$$\therefore \boxed{m = \frac{\frac{2P}{v} - \frac{1}{2}C_D\rho A v^2}{g}}$$

Now if $v = 250$ m/s $\Rightarrow P/v = 90$ kN

$\Rightarrow P = 90 \times 10^3 \times 250$ (W)

$= 2.25 \times 10^7$ W.

$= \boxed{22,500 \text{ kW}}$ or 22.5 MW

(b) So
$$m = \frac{\frac{2 \times 2.25 \times 10^7}{250} - \frac{1}{2} \times 0.05 \times 1.275 \times 2.0 \times 250^2}{9.81}$$

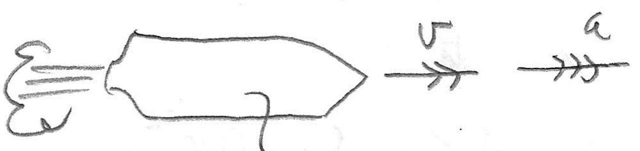
$$= \frac{180{,}000 - 3984.375}{9.81} \quad \text{(kg)}$$

$$= \boxed{17{,}940 \text{ kg}}$$

Wikipedia page.

{ Note numbers are loosely based upon a Eurofighter. Empty mass is 10,000 kg and fully loaded mass is about 21,000 kg. So it might not achieve this climb unless it loses about 3058 kg of mass. So for an airshow, don't add the bombs and missiles! }

(X)



$M = 12{,}000$ kg

$P = 2.0$ MJ/s

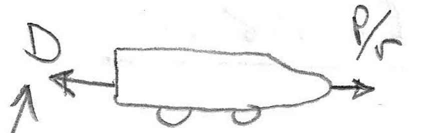
No drag in space, so power transferred to rocket by engines = rate of change of KE.

NII : $Ma = P/v$ $\therefore$ $\boxed{a = \frac{P}{Mv}}$

So if $P = 2.0 \times 10^6$ W, $M = 12{,}000$ kg
and $v = 10 \times 10^3$ m/s

$\Rightarrow$ $a = \dfrac{2.0 \times 10^6}{12000 \times 10 \times 10^3} = \boxed{1.7 \times 10^{-2}\ \text{m/s}^2}$

2/ a)



Drag $= \frac{1}{2} C_D \rho A v^2$

* Electric car of mass $M = 2{,}200$ kg
* Energy of fully charged lithium-ion cells $= E = 360$ MJ.
* Motor supplies max power of $P_{max} = 615$ kW

At constant speed $v = 31.3$ m/s (70 mph)

$P/v = \frac{1}{2} C_D \rho A v^2$ ie driving force = drag

$\therefore$ $P = \frac{1}{2} C_D \rho A v^3$

let max range be $r = vt$

Now $Pt = E$ So $r = \dfrac{vE}{\frac{1}{2} C_D \rho A v^3}$

$\Rightarrow t = E/P$

$\therefore$ $\boxed{r = \dfrac{E}{\frac{1}{2} C_D \rho A v^2}}$

{ Easier way of thinking about this:
work done against drag is $r \times \frac{1}{2} C_D \rho A v^2$.
This must $= E$ }

So $$r = \frac{360 \times 10^6}{\frac{1}{2} \times 0.1 \times 1.28 \times \underbrace{2.00 \times 1.44}_{A} \times 31.3^2}$$

$$\boxed{r = 1993 \text{ km}}$$

Impressive! Tesla Model 3 has a "long range" of 518 km, so assume more drag (and rolling resistance) and perhaps much less efficiency. ie E is electrical energy, and don't assume 100% efficiency.

So if 26% efficient $\Rightarrow$ $\boxed{r = 518 \text{ km}}$

ie $E = 0.26 \times 360 \times 10^6$
$= \boxed{93.6 \text{ MJ}}$

b) Now using $P = \frac{1}{2} C_D \rho A v^3$

$$\Rightarrow \quad v = \sqrt[3]{\frac{2P}{C_D \rho A}}$$

So if $P = P_{max} = 615 \times 10^3$ W

$$\Rightarrow \quad v_{max} = \sqrt[3]{\frac{2 \times 615 \times 10^3}{0.1 \times 1.28 \times 2.00 \times 1.44}}$$

$= 149$ m/s $= 149 \times \frac{70}{31.3}$ mph

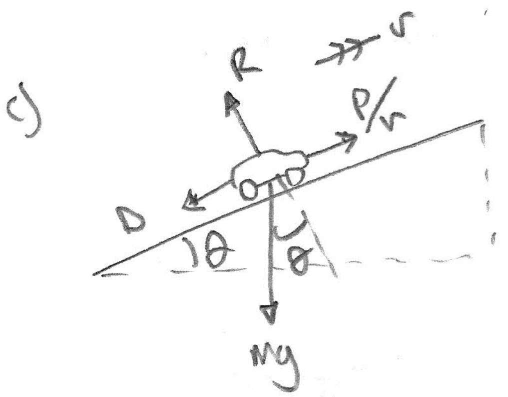
$= \boxed{334 \text{ mph}}$ (!)

This is rather on the high side! Probably rolling resistance is significant (ie can't be ignored) and also 615 kW may not = Driving force × velocity. If 26% efficiency is used, $v_{max} = \sqrt[3]{0.26} \times 334$ mph

$= \boxed{213 \text{ mph}} = 95.2 \text{ m/s}$

which is probably more realistic for a high performance car.

[Top speed for a Tesla Model 3 is ≈ 160 mph.]

c) 

$g = 9.81$ N/kg

NII // slope:

$$0 = P/v - D - mg\sin\theta$$

so $$\underbrace{\tfrac{1}{2}C_D\rho A v^2}_{D} + mg\sin\theta - P/v = 0$$

Not easy to solve the cubic for v, so let $v \to \alpha v$ where $\alpha = 0.9$.

$$\boxed{\theta = \sin^{-1}\left(\frac{P/v - \tfrac{1}{2}C_D\rho A v^2}{mg}\right)}$$

Now from above: $P_{max} = \tfrac{1}{2}C_D\rho A v_{max}^3$ and let $P = P_{max}$

$$\theta = \sin^{-1}\left(\frac{P - \tfrac{1}{2}C_D\rho A v^3}{mgv}\right)$$

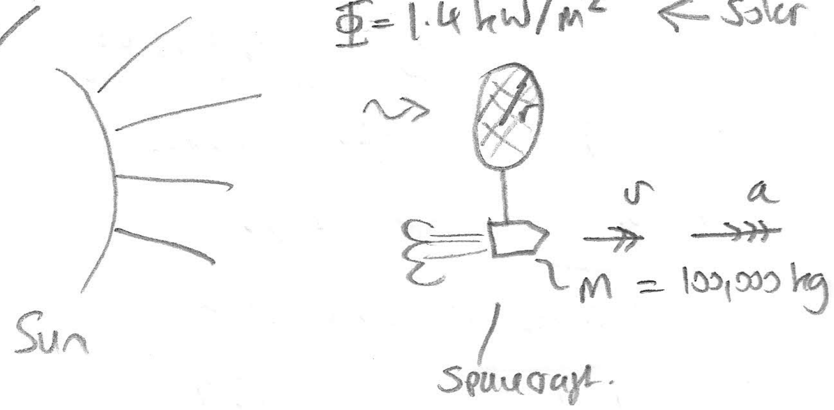
$$\therefore \theta = \sin^{-1}\left(\frac{P_{max} - \tfrac{1}{2}C_D\rho A v_{max}^3\alpha^3}{mg v_{max}\alpha}\right) = \sin^{-1}\left(\frac{P_{max}}{mg v_{max}}\left(\frac{1-\alpha^3}{\alpha}\right)\right)$$

$$\therefore \theta = \sin^{-1}\left(\frac{615\times 10^3}{2200\times 9.81\times 95.2}\left(\frac{1-0.9^3}{0.9}\right)\right) = \boxed{5.2^\circ}$$

3/

$\Phi = 1.4\,kW/m^2$ ← Solar 'radiation flux'

Conversion of light energy efficiency $\eta = 0.5$.



[Note you can get $\boxed{\text{radiation pressure}}$ from solar photons

Since $\underbrace{hf}_{\text{photon energy}} = pc$ p is photon momentum, c is speed of light.

So a 'solar sail' can be used in the non-electrical sense too]

Newton II: $$Ma = \frac{\eta \Phi \times \pi r^2}{v}$$

Now $a = \frac{dv}{dt}$ so $$\frac{v\,dv}{dt} = \eta \Phi \pi r^2/M$$

$$\therefore \int_0^v v\,dv = \eta \Phi \pi r^2/M\; t$$

$$\frac{1}{2}v^2 = \eta \Phi \frac{\pi r^2}{M} t$$

{ or do $\frac{1}{2}Mv^2 = \eta \Phi \pi r^2 t$ }

ie KE gain = energy absorbed.

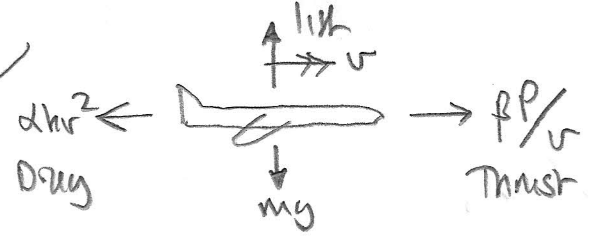
$$\therefore \boxed{\sqrt{\frac{Mv^2}{2\eta \Phi \pi t}} = r}$$

$$\therefore r = \sqrt{\frac{100000 \times (20 \times 10^3)^2}{2 \times 0.5 \times 1400 \times \pi \times 3600}}$$

$$\boxed{r = 1590\text{ m}}$$

(which is huge on Earth scales, but not so crazy in space as it is a near vacuum).

4/



$M = 250 \times 10^3$ kg

Altitude /m	Drag α parameter
0	1
6,000	0.55
12,000	0.30

(i) $\boxed{\text{Horizontal flight}}$ {lift = weight}
(constant speed)

Newton II: let $\beta = 1$
ie power P is cruising power

$$0 = P/v - \alpha k v^2 \Rightarrow \boxed{P = \alpha k v^3}$$

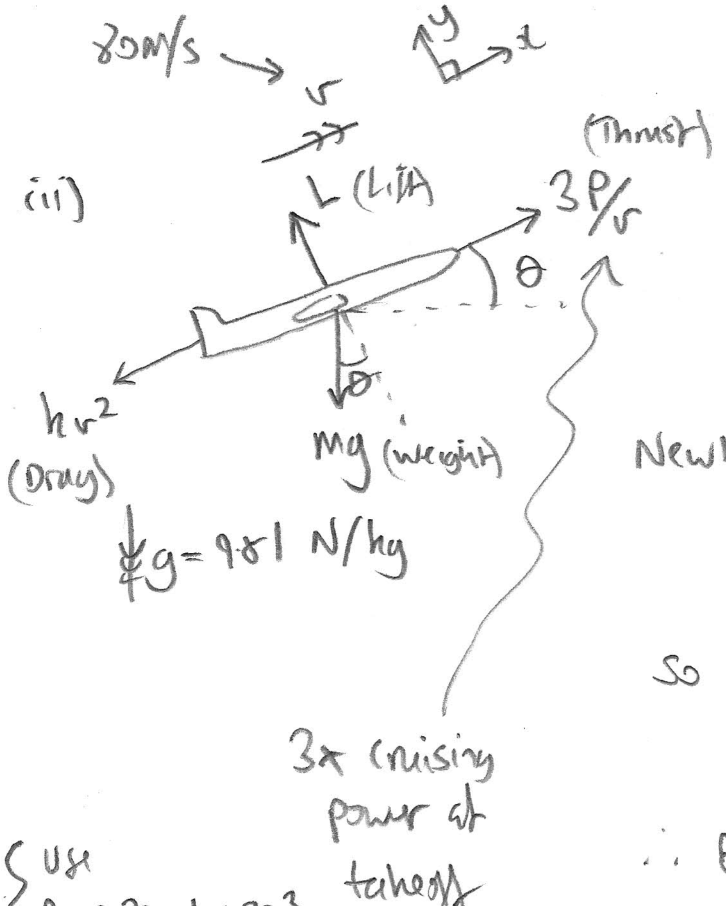
So $P = 0.30 \times k \times 220^3$ ① (12,000m altitude)
$P = 0.55 \times k \times v^3$ ② (6,000m altitude)

②/① $\left(\frac{v}{220}\right)^3 \frac{0.55}{0.3} = 1$ $\therefore v = 220\,\text{m/s} \times \sqrt[3]{\frac{0.3}{0.55}}$

$v = 220\,\text{m/s} \times 0.817$

$\boxed{v = 180\,\text{m/s}}$

(ii)



ie at takeoff $\alpha = 1$
let $k = 2.5\,N\,(m/s)^{-2}$
$M = 250 \times 10^3$ kg

Newton II: $/\!/x: 0 = 3P/v - kv^2 - Mg\sin\theta$

$/\!/y: 0 = L - Mg\cos\theta$

So $\sin\theta = \frac{3P/v - kv^2}{Mg}$

$\therefore \theta = \sin^{-1}\left(\frac{3P/v - kv^2}{Mg}\right)$

{use $P = 0.30 \times k \times 220^3$ from above}

$= \sin^{-1}\left(\frac{\frac{0.30 \times 2.5 \times 220^3 \times 3}{80} - 2.5 \times 80^2}{250 \times 10^3 \times 9.81}\right) = \boxed{6.6°}$

Now $L = mg\cos\theta$

$\therefore L = 250 \times 10^3 \times 9.81 \times \cos(\theta)$

$= \boxed{2436\ kN}$

AP
1/7/2020.