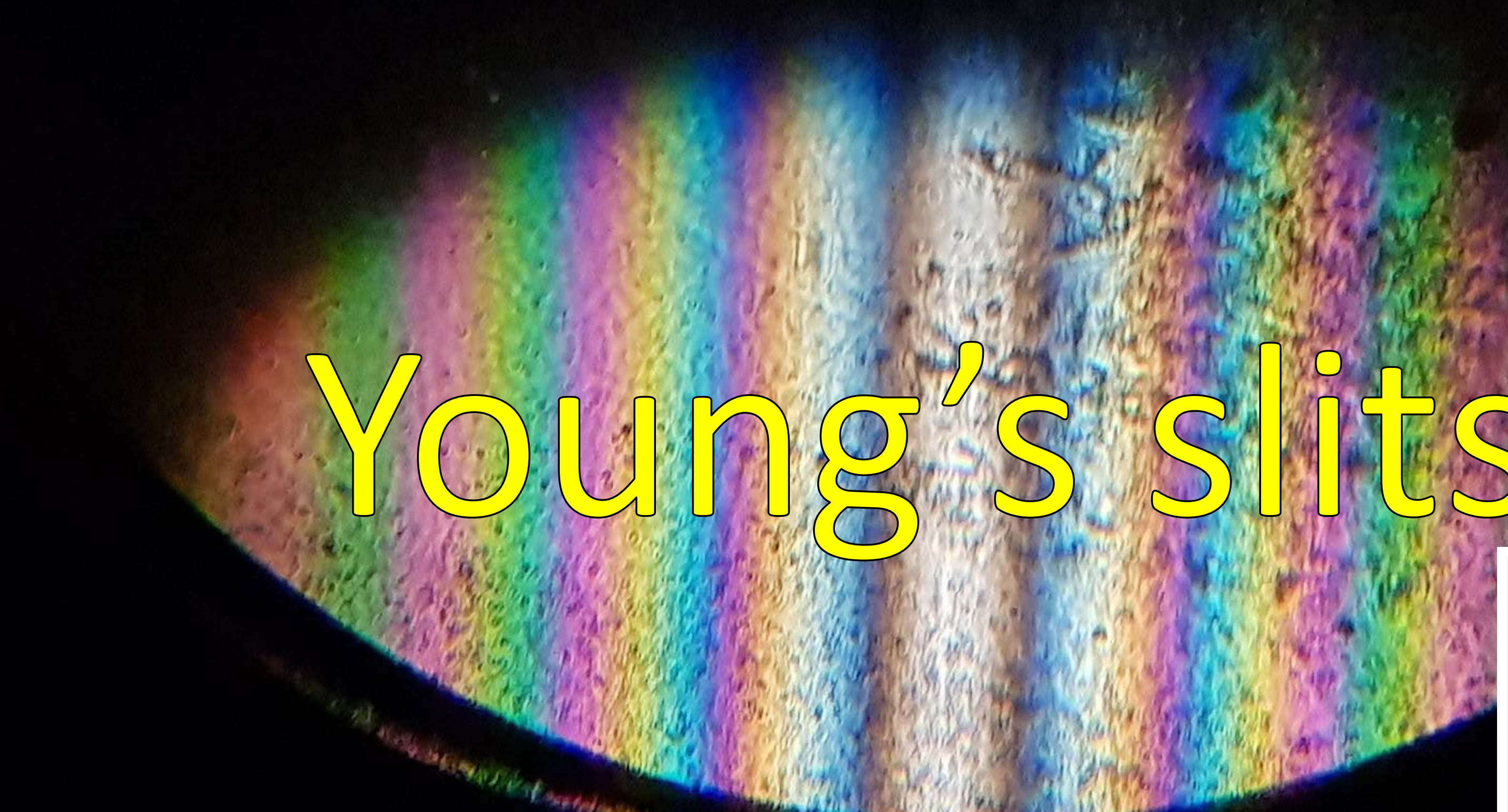
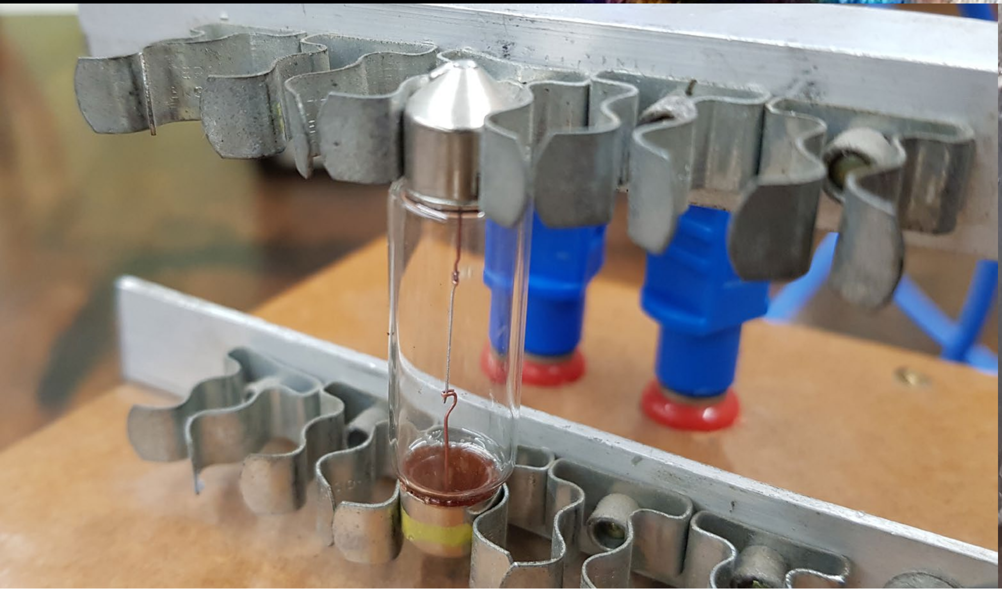
Young’s slits



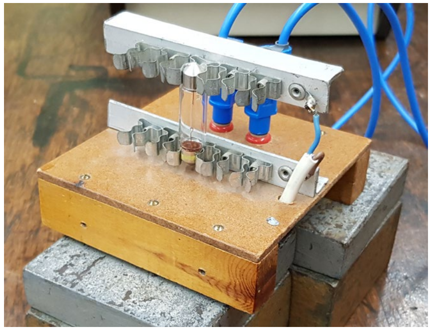
Thomas Young
1773-1829

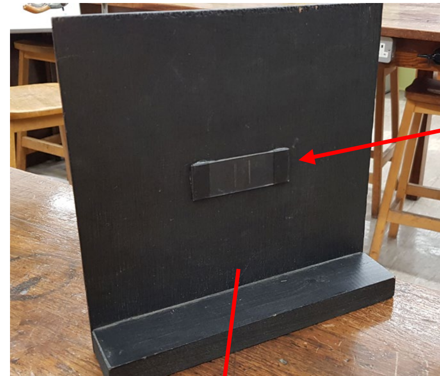


AF. May 2024.

Equipment setup

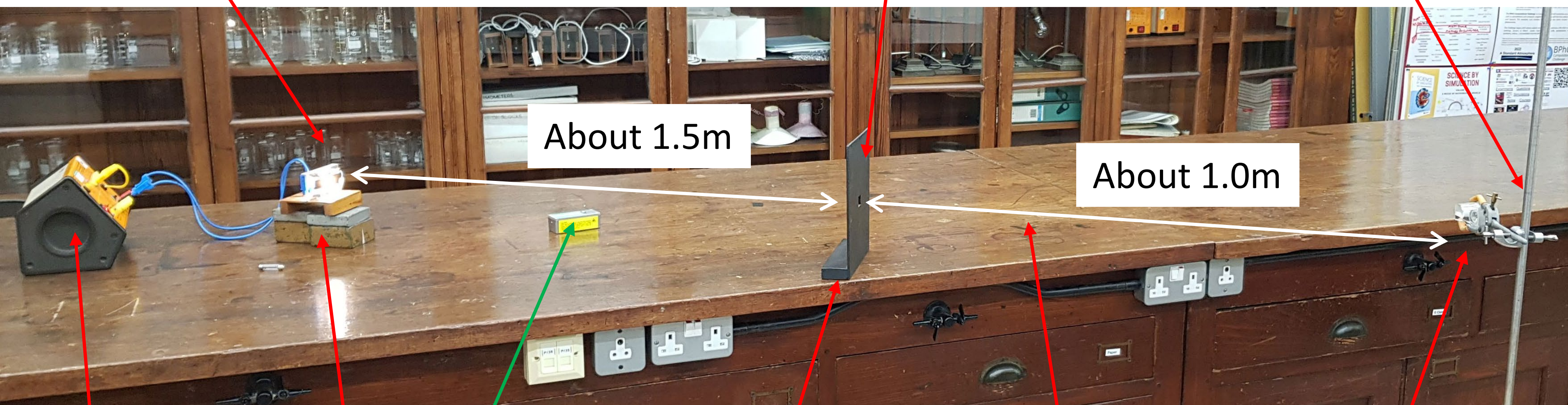
Vertical filament bulb kit





Double slits

Retort stand, boss and clamp



DC power supply (10V)

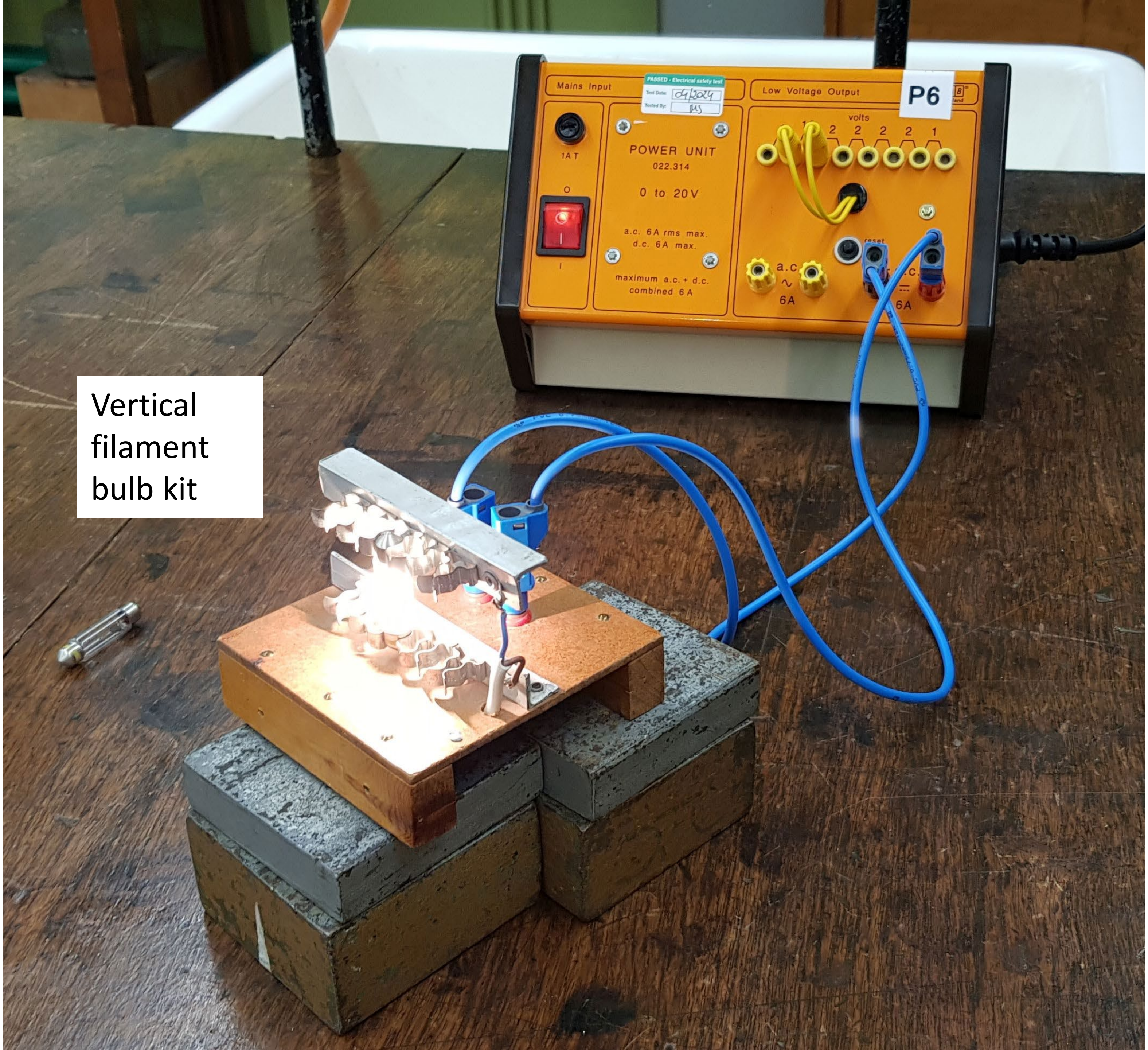
Blocks

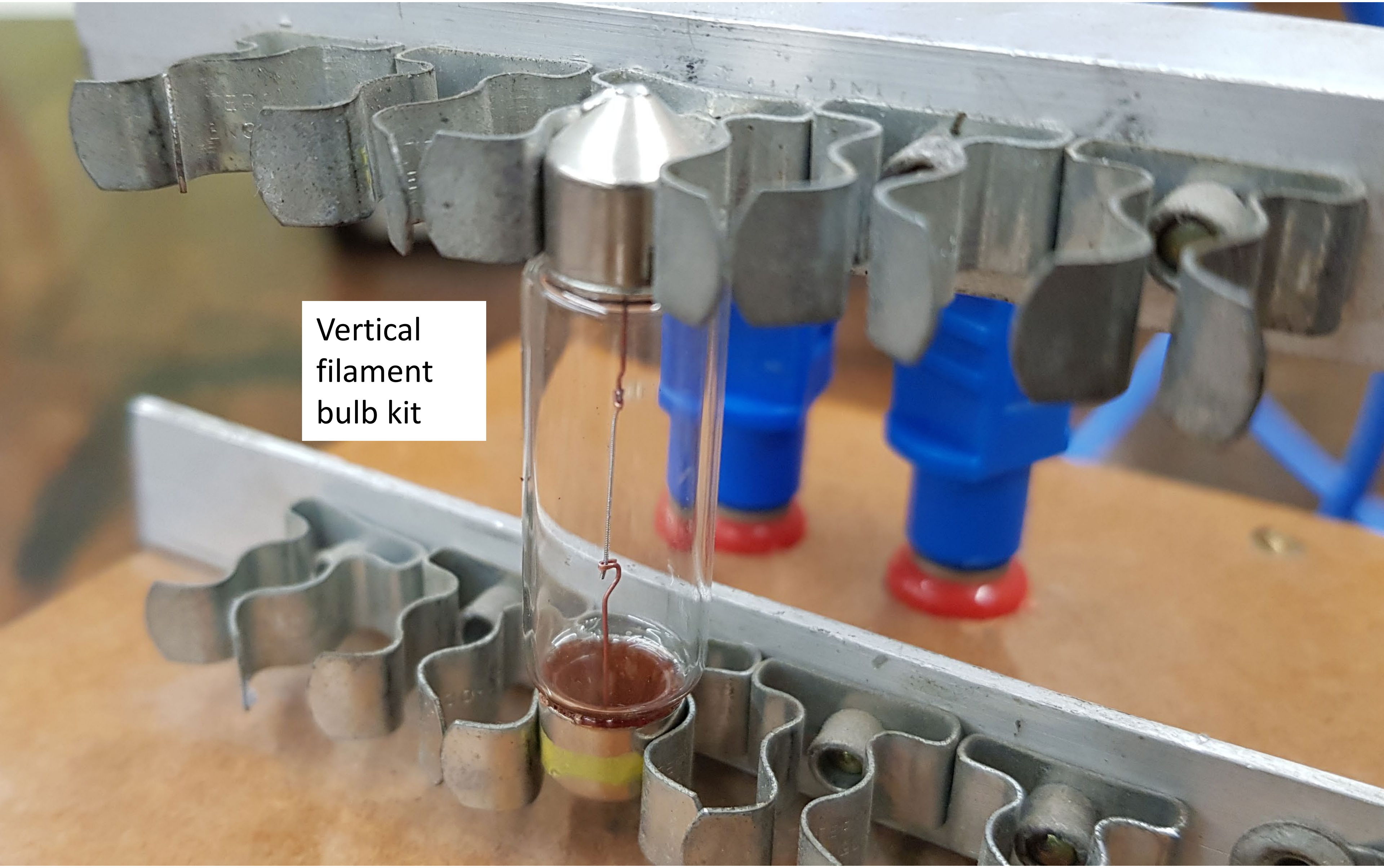
Mounting board for Young's double slits

Magnification Eyepiece

Green laser (a follow-on experiment)

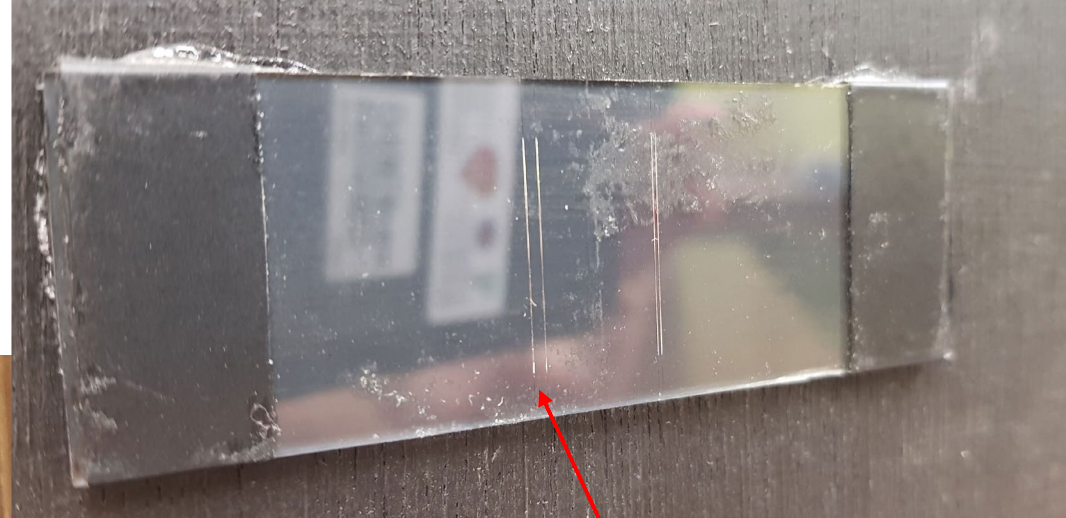
Measure slit to eyepiece displacement accurately (with a metre rule) once you have successfully located coloured fringes in the eyepiece.

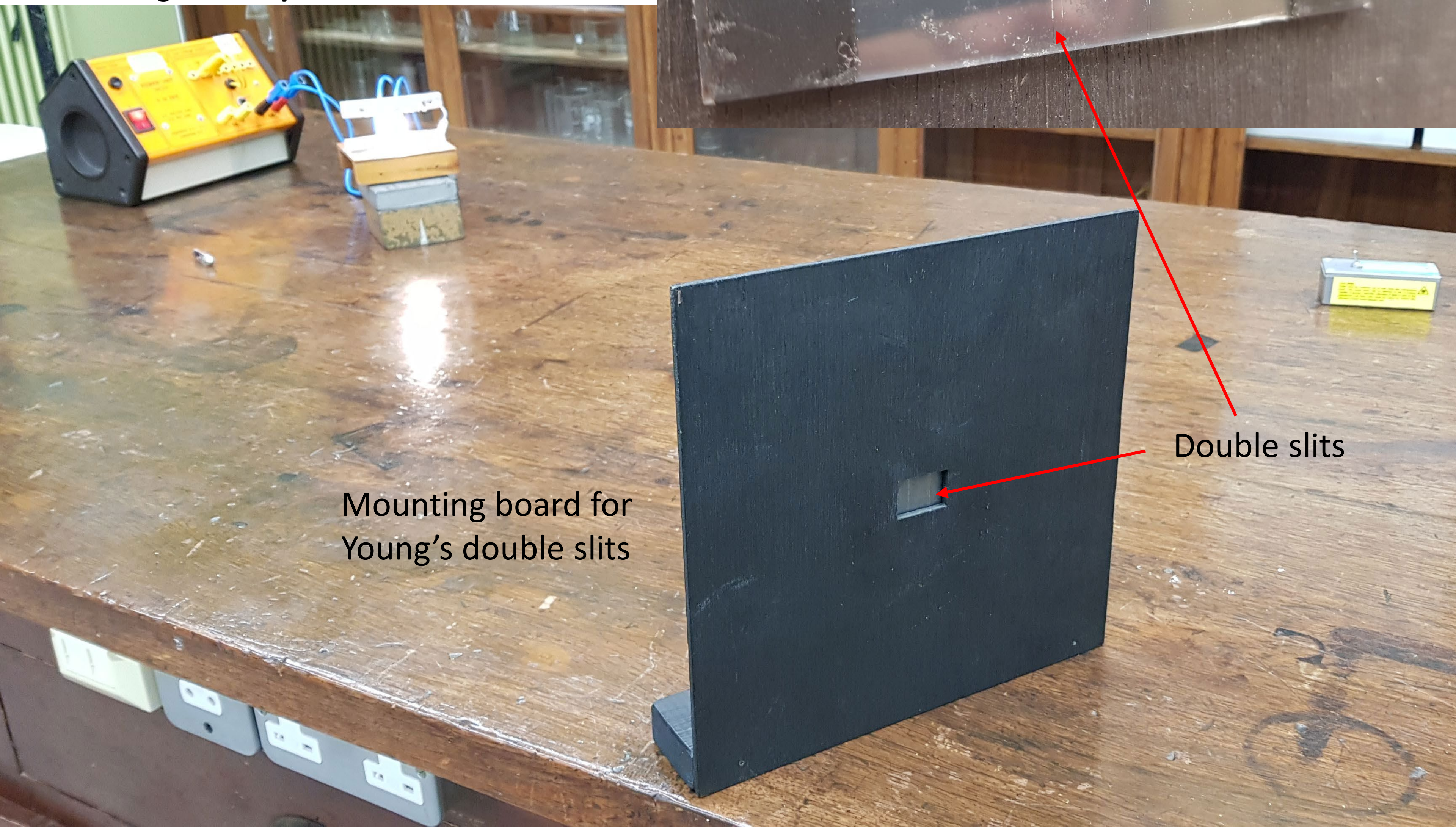

Vertical filament bulb kit
Mains Input
Low Voltage Output
P6
POWER UNIT
022.314
0 to 20V
a.c. 6A rms max.
d.c. 6A max.
maximum a.c. + d.c.
combined 6A
volts
a.c
6A

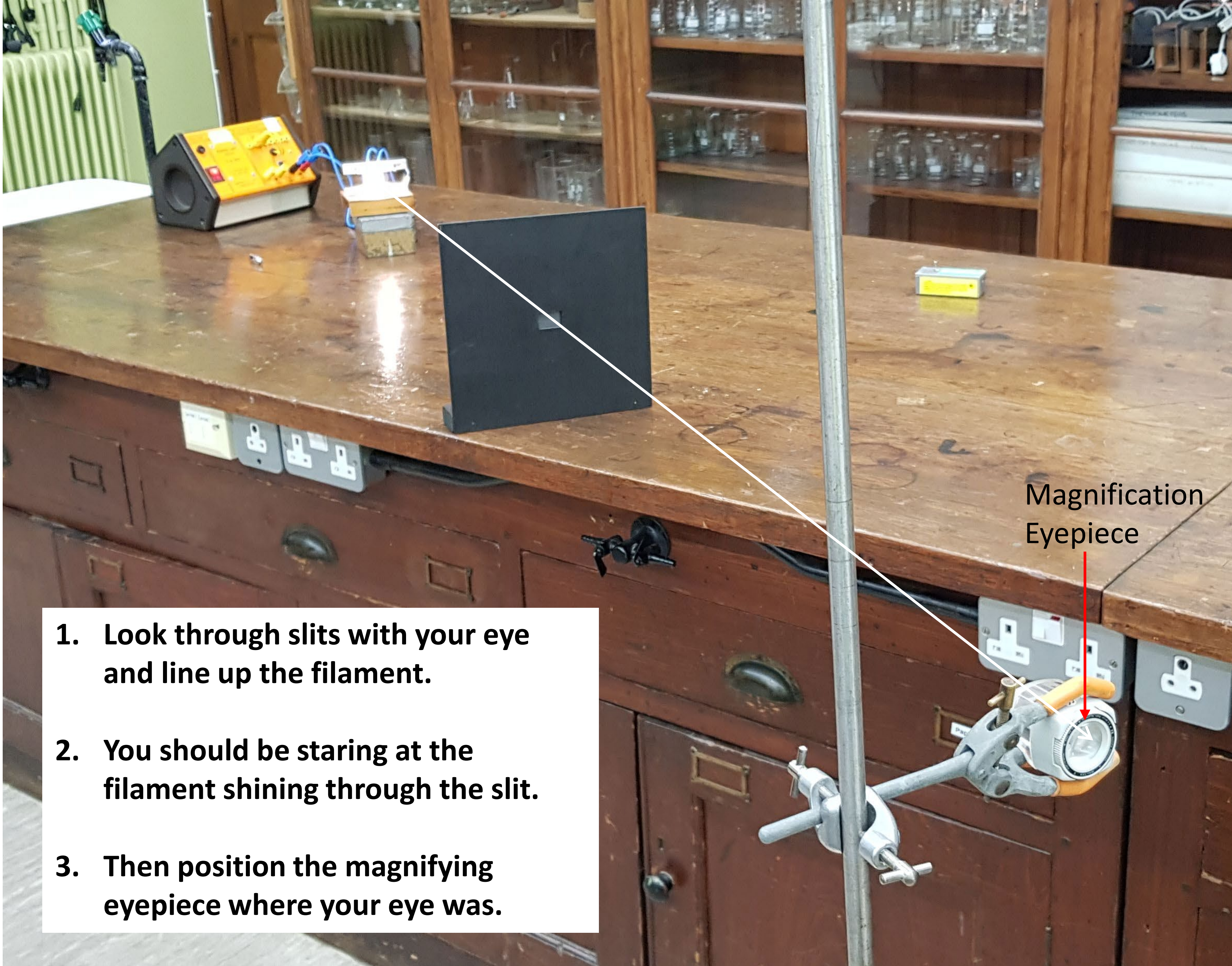

Vertical filament bulb kit

Each board has a different pair of double slits. Spacing varies from about 1 to 5mm.

Measure spacing precisely with a **digital caliper**.







1. Look through slits with your eye and line up the filament.

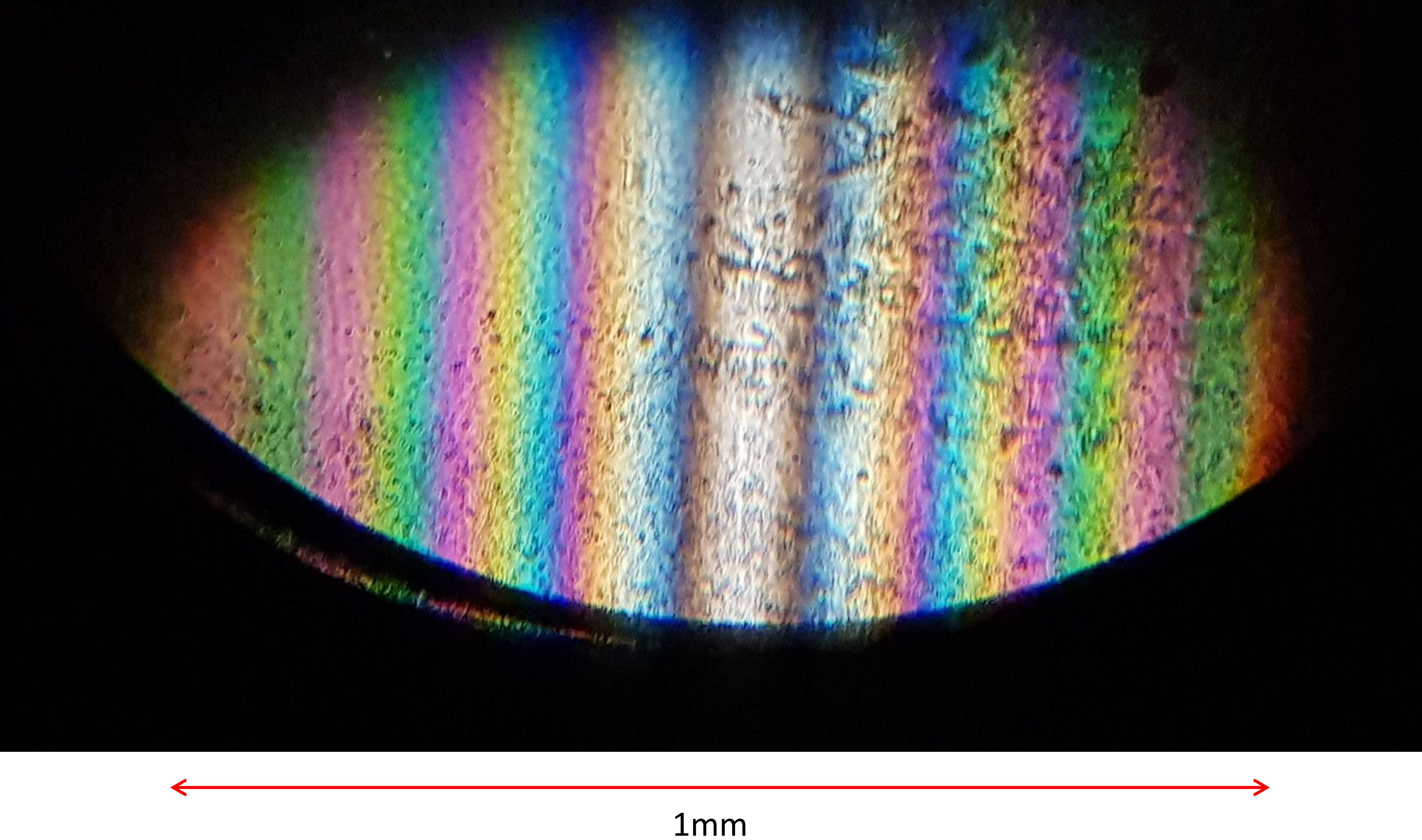
2. You should be staring at the filament shining through the slit.

3. Then position the magnifying eyepiece where your eye was.

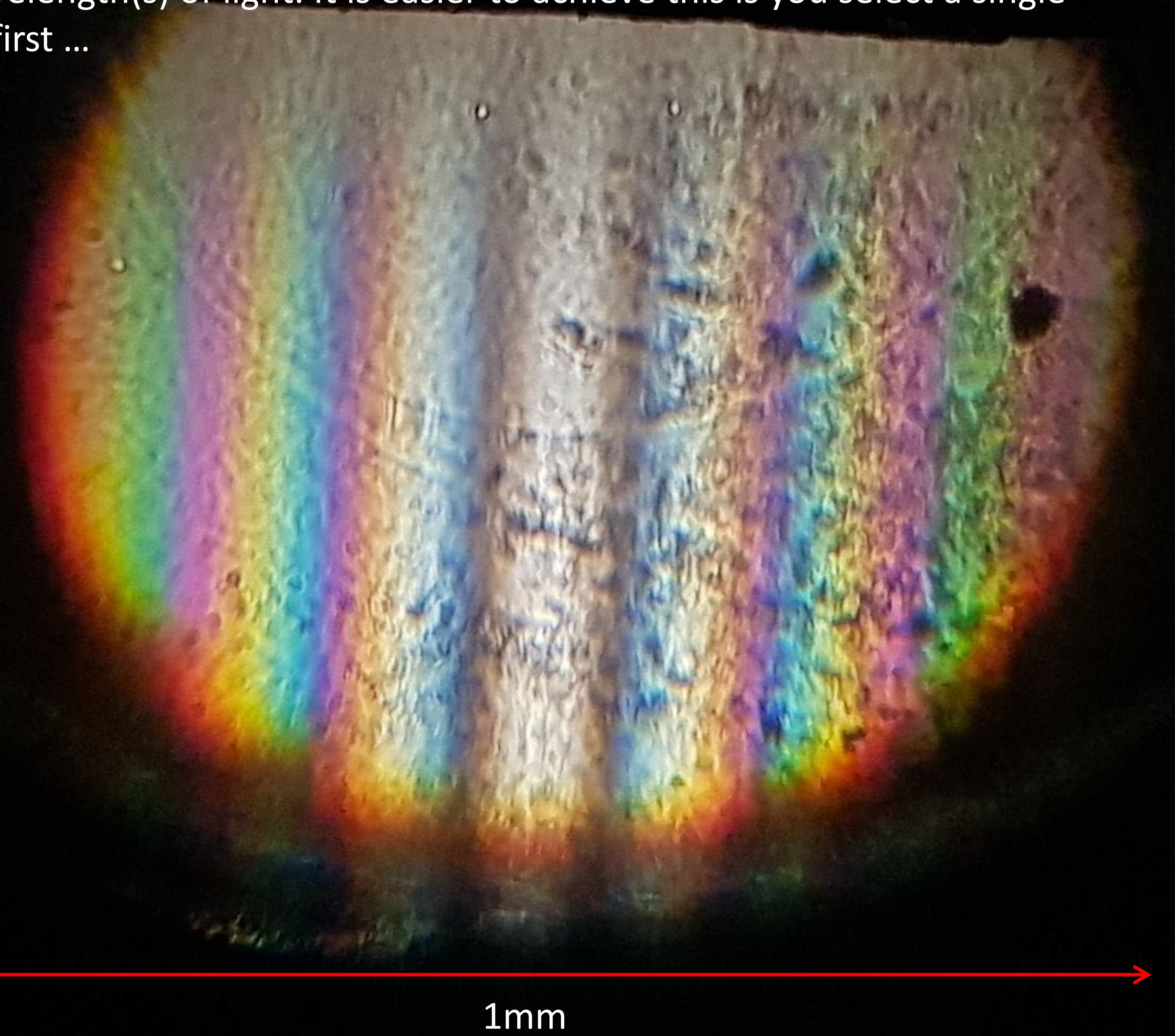
Stare though the eyepiece, and hopefully you will see coloured fringes formed by the interference of light emerging from the double slit….

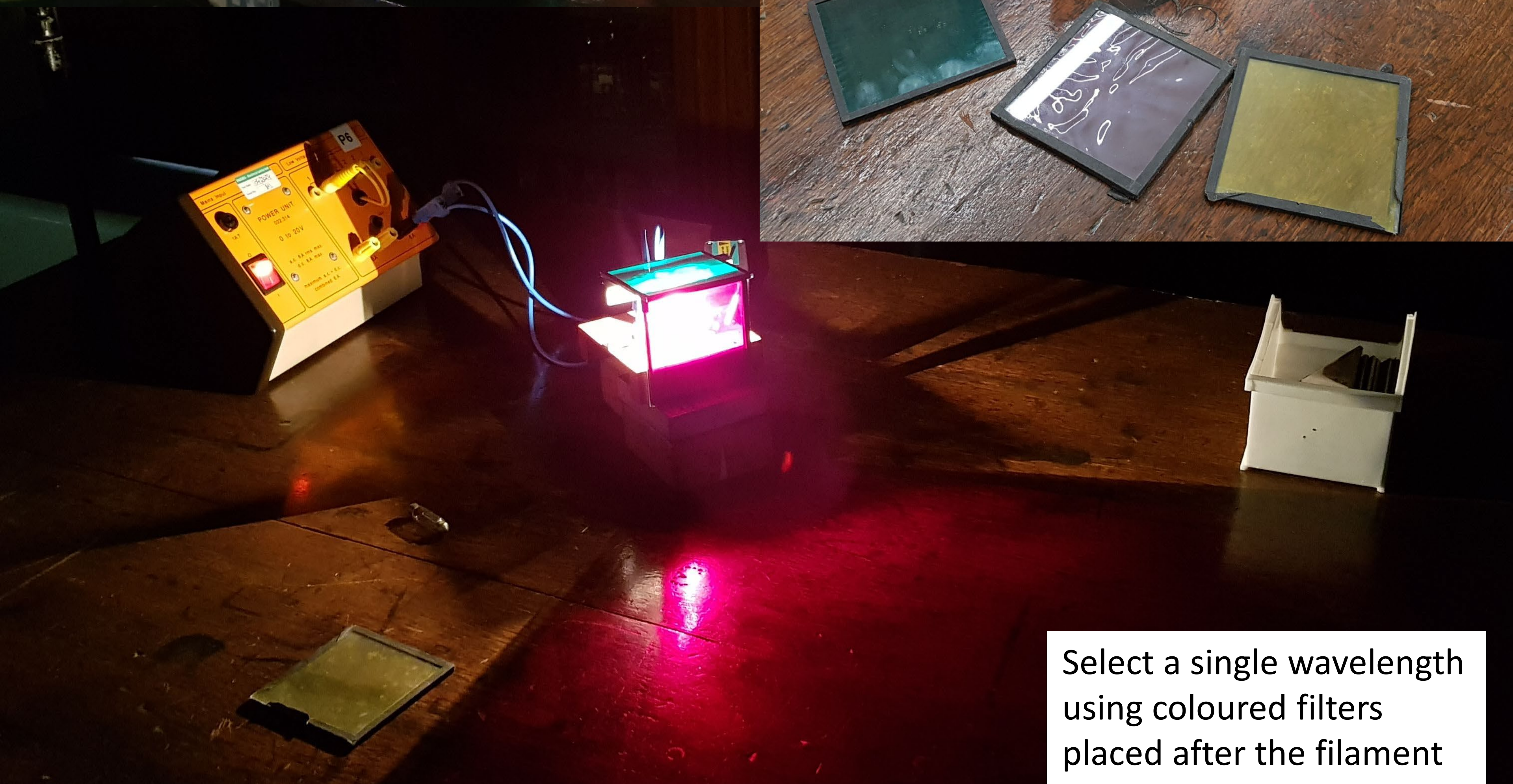
Note scale in field of view of eyepiece is **1mm** (divided up into 100 lines)

Fringes resulting from the interference of the red to purple wavelengths of light passing through the pair of thin slits of spacing s.



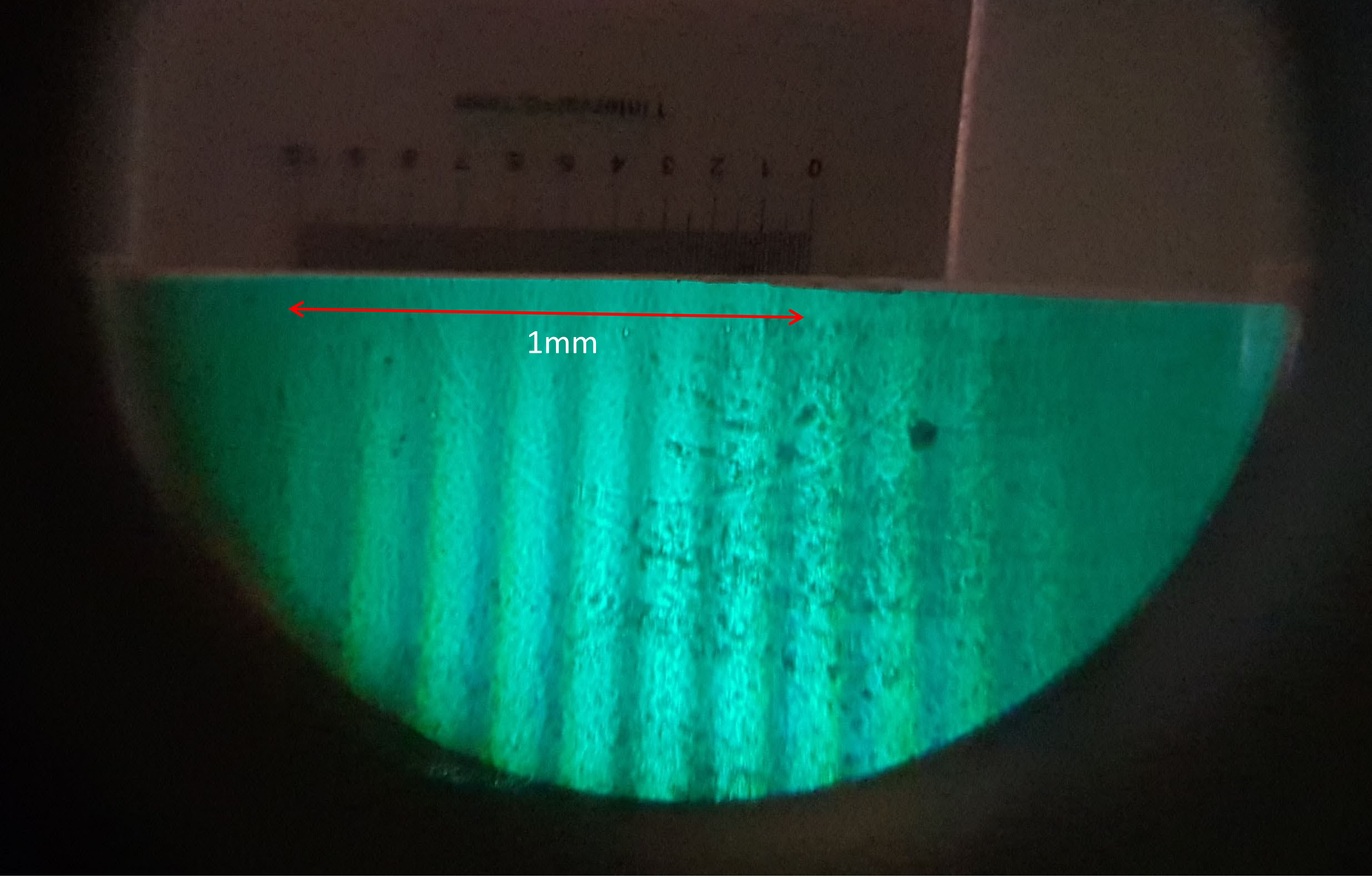
Count the fringes relative to the 1mm scale and hence determine the wavelength(s) of light. It is easier to achieve this is you select a single colour first …

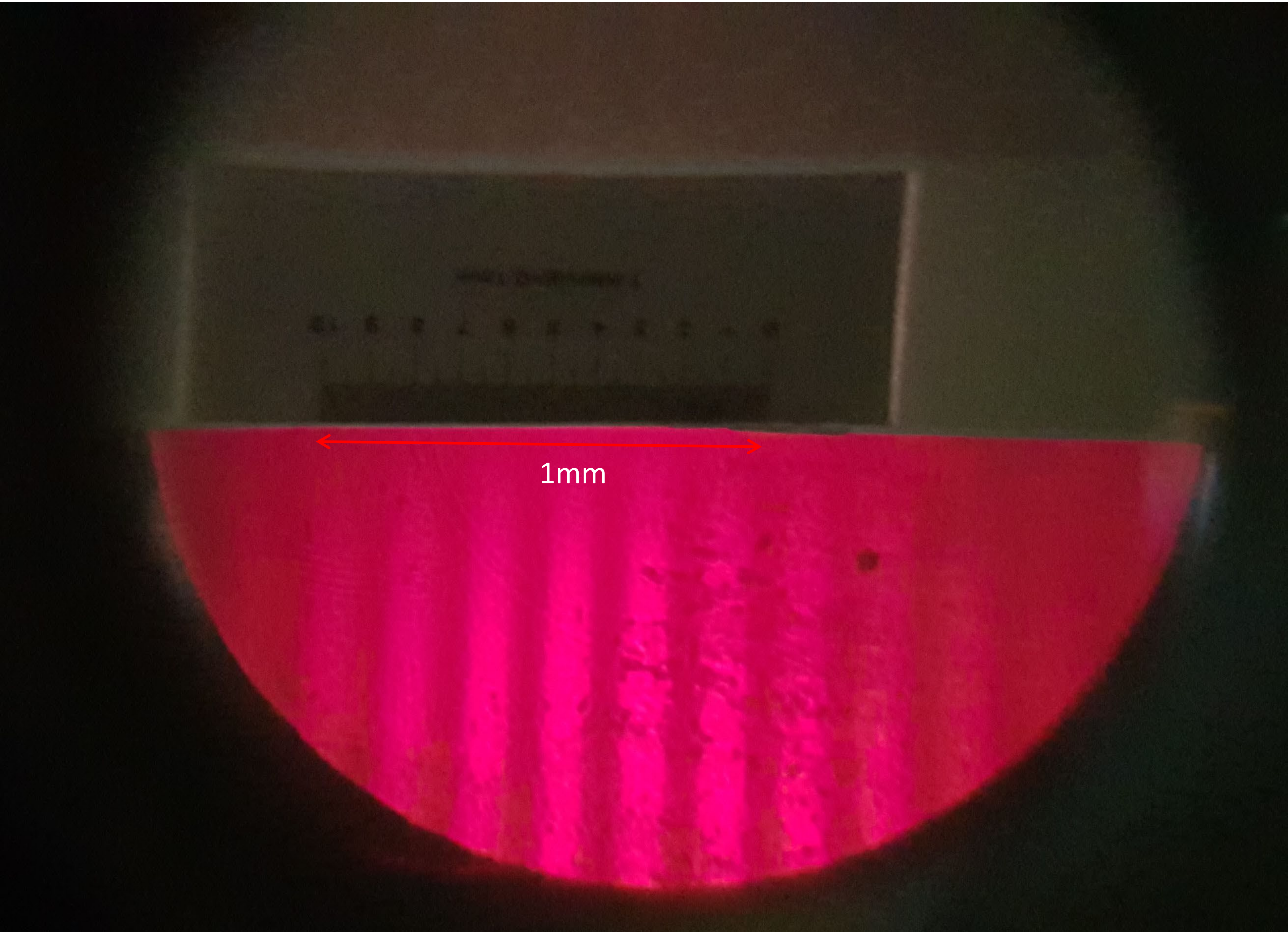


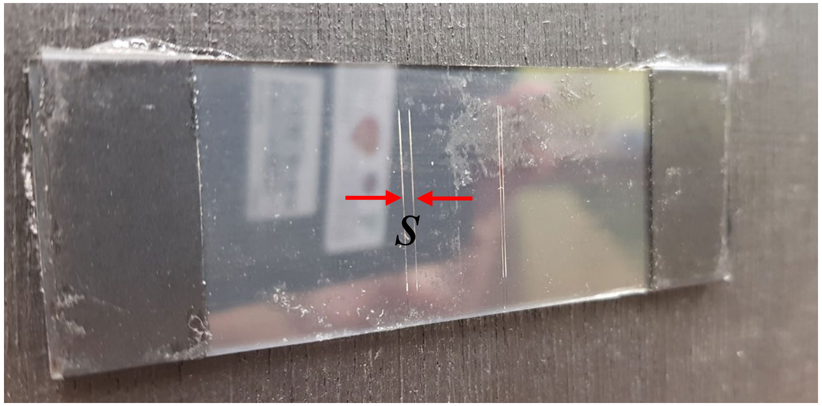


Select a single wavelength using coloured filters placed after the filament

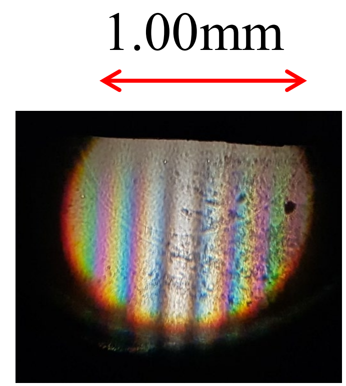
Count the fringes relative to the 1mm scale and hence determine the wavelength(s) of light. It is easier to achieve this is you select a single colour first …

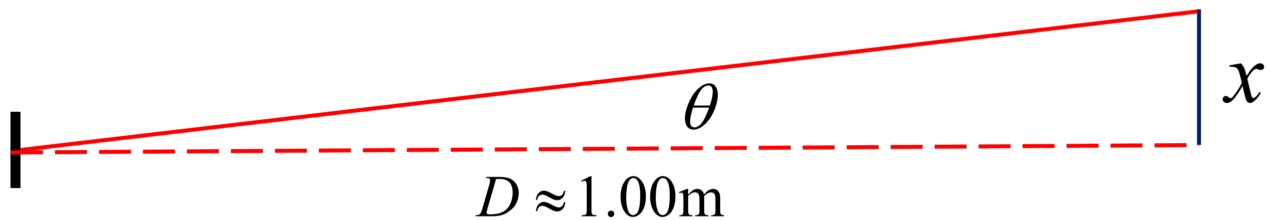



1mm



Young's double slits of spacing s and slit width w





Young's slits

wavelength

$$s \sin\theta = n\lambda$$

integer

Fringe maxima
(constructive interference)

$$x = D\tan\theta$$

From geometry of eyepiece and slit

Note since angles are *small* $x \approx D\theta \approx D\sin\theta \approx \dfrac{Dn\lambda}{s}$

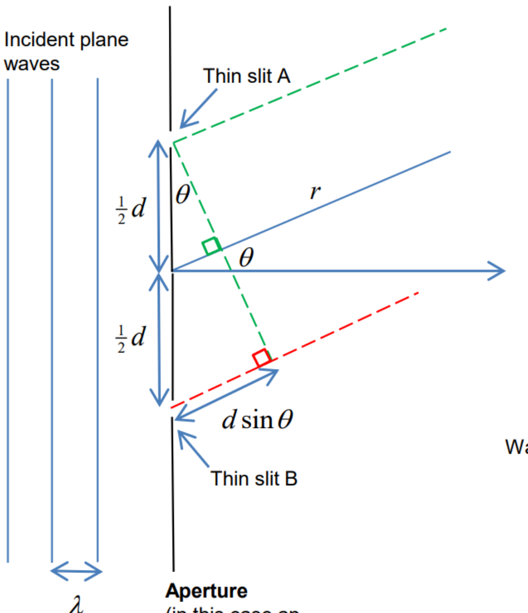
Wavenumber $k = \frac{2\pi}{\lambda}$ $\omega = 2\pi f$ Frequency Wave speed $c = f\lambda$, $\omega = ck$ Wave power input $P \propto A^2\omega^2$

Diffraction Essentials

The ***Huygens-Fresnel Principle*** states: *"Every unobstructed point of a wavefront, at a given instant, serves as a source of spherical secondary wavelets (with the same frequency as that of the primary wave). The amplitude of the optical field at any point beyond is the superposition of all these wavelets (considering their amplitudes and relative phases)."*

Hence to determine the *wavefield* beyond an illuminated edge of a slit, we need to add up the effect of spherical wave sources in the vicinity of the slit or *aperture*.

Key geometrical idea from two infinitesimally thin slits ('Young's Slits')



Aperture
(in this case an opaque wall with two infinitesimally thin slits)

Spherical waves will emanate from the slits, and interfere with each other.

For distances such that: $r \gg \frac{d^2}{\lambda}$
(we call this the **Far Field**)
we can assume waves from each slit are **plane waves**, for any given observational angle θ.

Constructive interference occurs when the *phase difference* between the waves from slits A and B is an integer multiple of 2π radians.

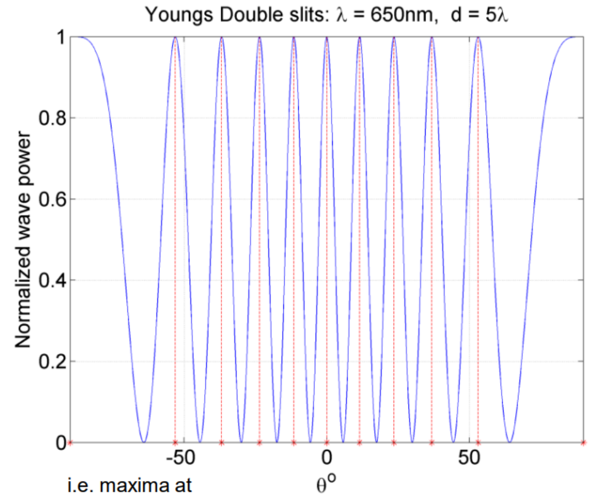
$$\frac{2\pi}{\lambda} d\sin\theta = 2\pi n$$

Wavenumber k — *Path difference* between waves from A and B — Integer n

Hence expect **maxima** in the resulting **Far Field Diffraction pattern** (e.g. spots of a laser on a wall) at angles

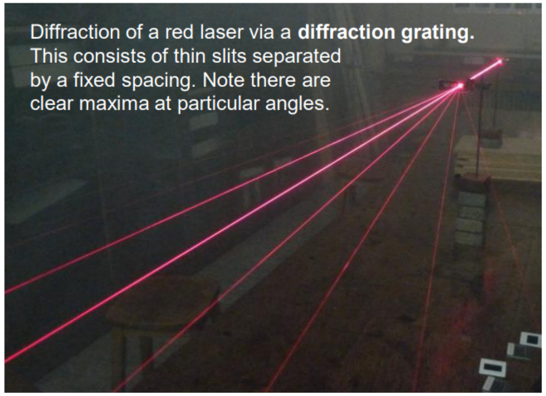
$$\sin\theta = \frac{n\lambda}{d}$$

Since the diffraction angle is *inversely* related to spacing d we can use **diffraction patterns** to *measure* small periodic structures (e.g. atomic layers, structure of DNA...) in the laboratory!



i.e. maxima at

$$\theta = \sin^{-1}\left(\tfrac{1}{5}n\right) = 0°, \pm 11.5°, \pm 23.6°, \pm 36.9°, \pm 53.1°, \pm 90°$$



Diffraction of a red laser via a **diffraction grating.** This consists of thin slits separated by a fixed spacing. Note there are clear maxima at particular angles.

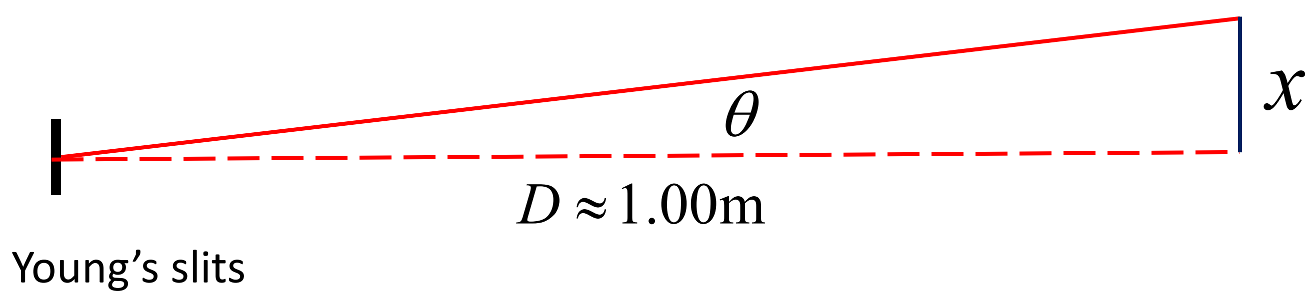
Christiaan Huygens
1629-1695

Thomas Young
1773-1829

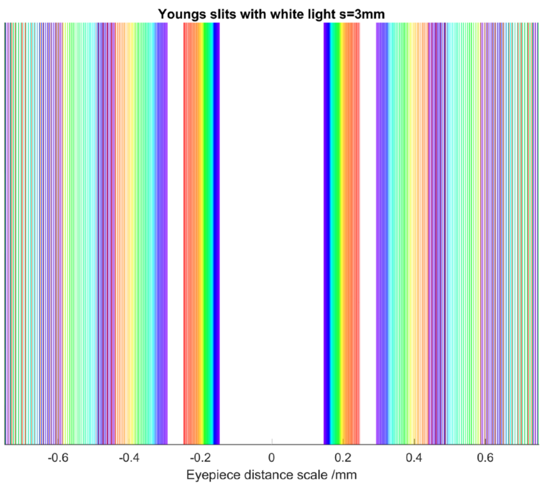
Augustin-Jean Fresnel
1788-1827

```
%youngs_slits_fringes
% Simulates the coloured fringe pattern produced from a white light source
% which illuminates a pair of (thin) double slits of spacing s and width w.
% It is assumed all spectral components contribute equally to the resulting
% image, over the range of frequencies 405THz (red) to 680THz (Purple).
%
% LAST UPDATED by Andy French May 2024.

function youngs_slits_fringes

D = 1.00;  %Distance from slit to magnifying eyepiece (in m).
c = 2.998e8;  %Speed of light /ms^-1
s = 10; %Slit spacing /mm
w = 0.1; %Slit width /mm
xw = 1.5; %Width of (magnifying) eyepiece scale
P = 5000; %Data points for eyepiece scale plot

%

%Define an array of frequencies /THz which comprise the white light source
%incident to the double slit
f = linspace(405,680,50);

%Compute wavelengths /nm
lamda = 1e9*c./(f*1e12);
```

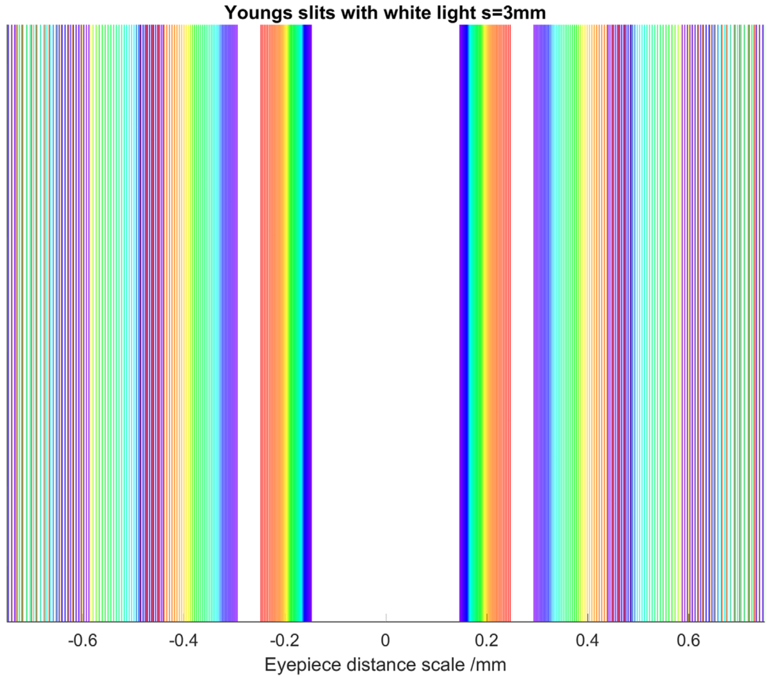
```matlab
%Determine distance along magnified scale (-xw/2 ... xw/2)
%(in mm) corresponding to light of wavelength lamda (in nm)
%illuminating the double slit.
function x = fringes( lamda, D, s, xw )
n=0; x=0;
while x(end)<xw/2
    n = n+1;
    %Angle of fringe maxima /rad
    theta = asin( n*lamda*(1e-9)/(s*1e-3) );
    %Fringe distance in frame of eyepiece /mm
    x = [x,D*1000*tan(theta)];
end

%Add negative n values to make the fringe pattern symmetric
xx = fliplr(x); x = [-xx(1:end-1),x];
```

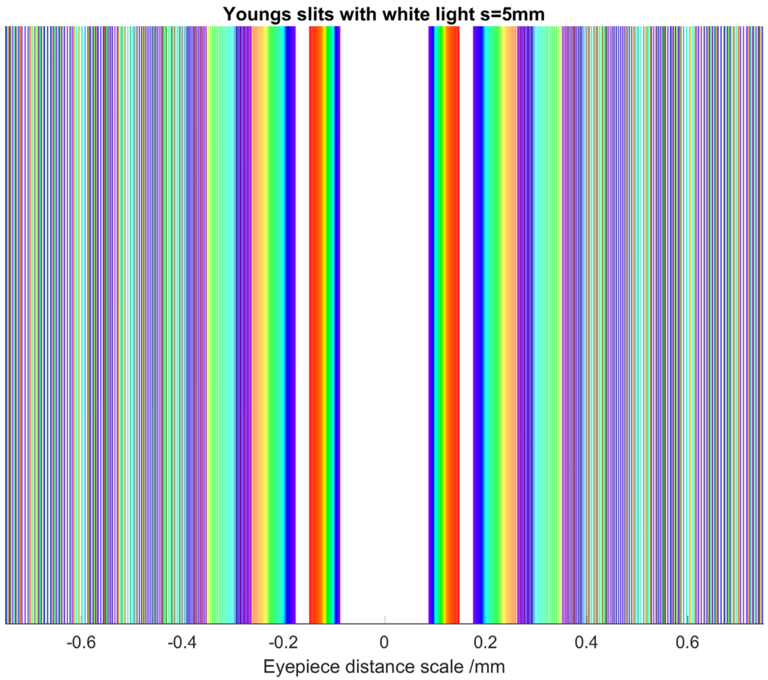


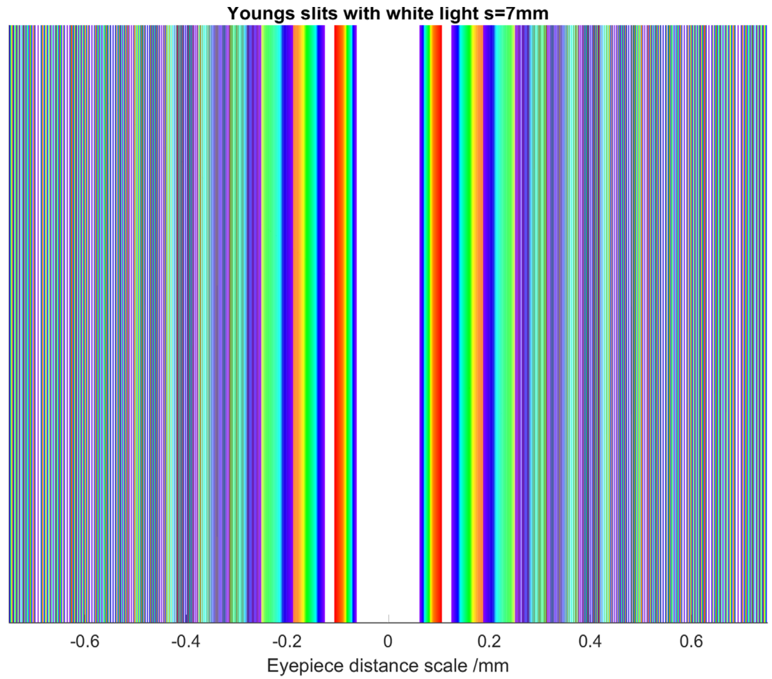
```
%RGB colour from light frequency /THz
function RGB = RGB_from_f(f)
F = [405,480,510,530,600,620,680];
R = [1,1,1,0,0,0,137/255];
G = [0,127/255,1,1,1,0,0];
B = [0,0,0,0,1,1,1];
RGB = zeros( numel(f),3 );
r = interp1( F,R,f ); g = interp1( F,G,f );
b = interp1( F,B,f );
RGB(:,1) = r(:); RGB(:,2) = g(:); RGB(:,3) = b(:);
```

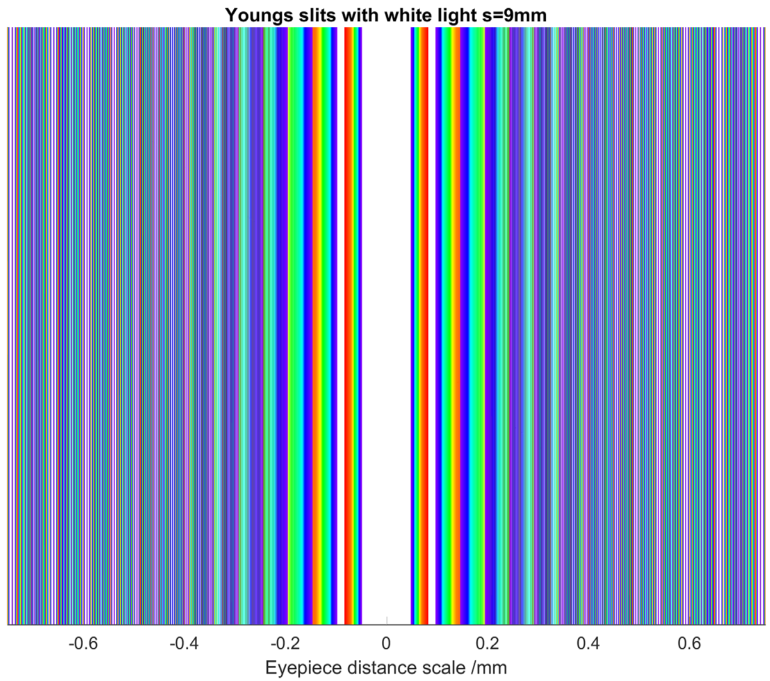



```
%Plot vertical lines for each fringe
for n=1:length(f)
    x = fringes( lamda(n), D, s, xw );
    for m=1:length(x)
        plot( x(m)*[1,1],[0,1],'color',RGB_from_f(f(n) ) );
    end
end
plot( [0,0],[0,1],'color',[1 1 1] );
```


Youngs slits with white light s=3mm
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
Eyepiece distance scale /mm


Youngs slits with white light s=5mm
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
Eyepiece distance scale /mm

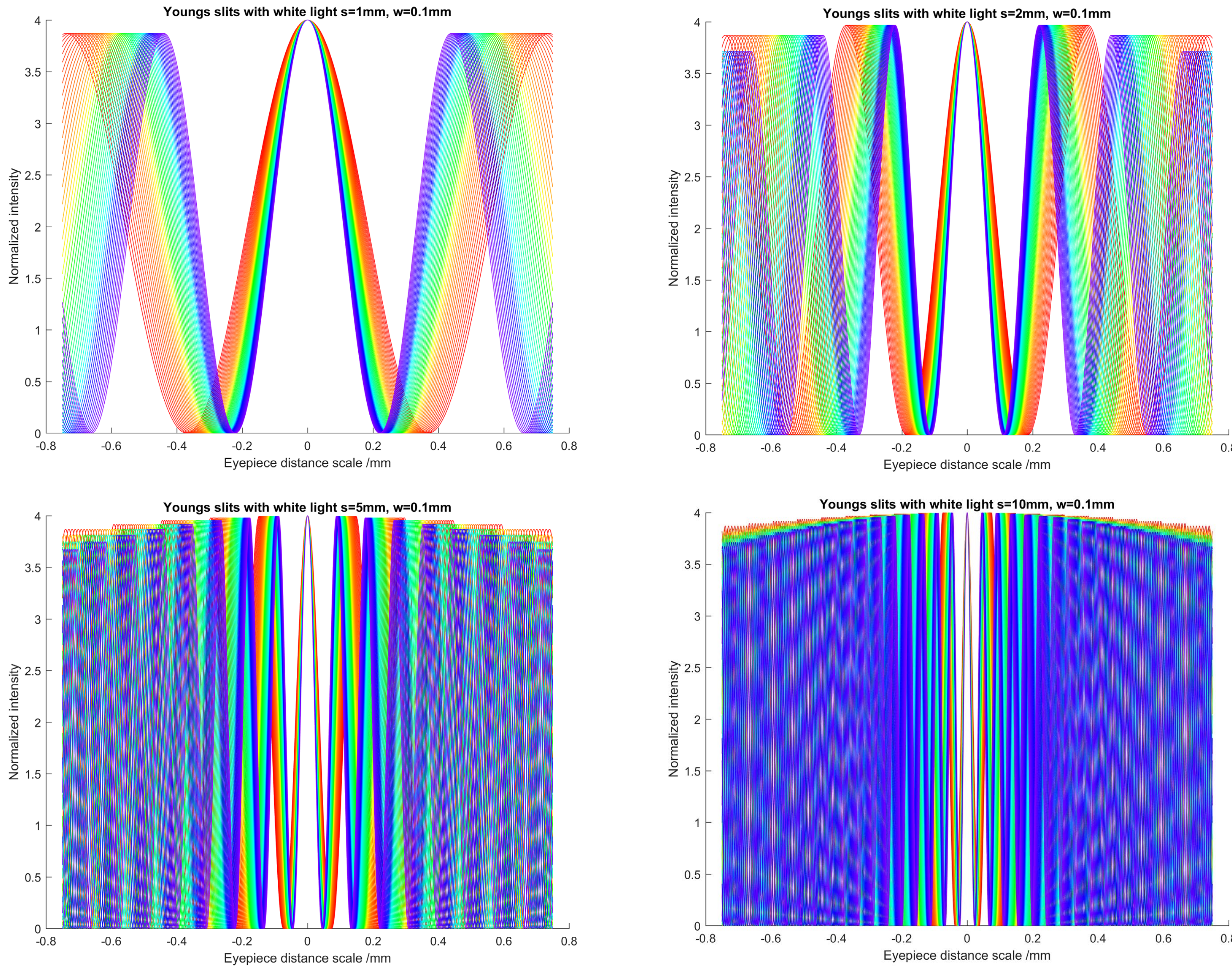

Youngs slits with white light s=7mm
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
Eyepiece distance scale /mm


Youngs slits with white light s=9mm
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
Eyepiece distance scale /mm

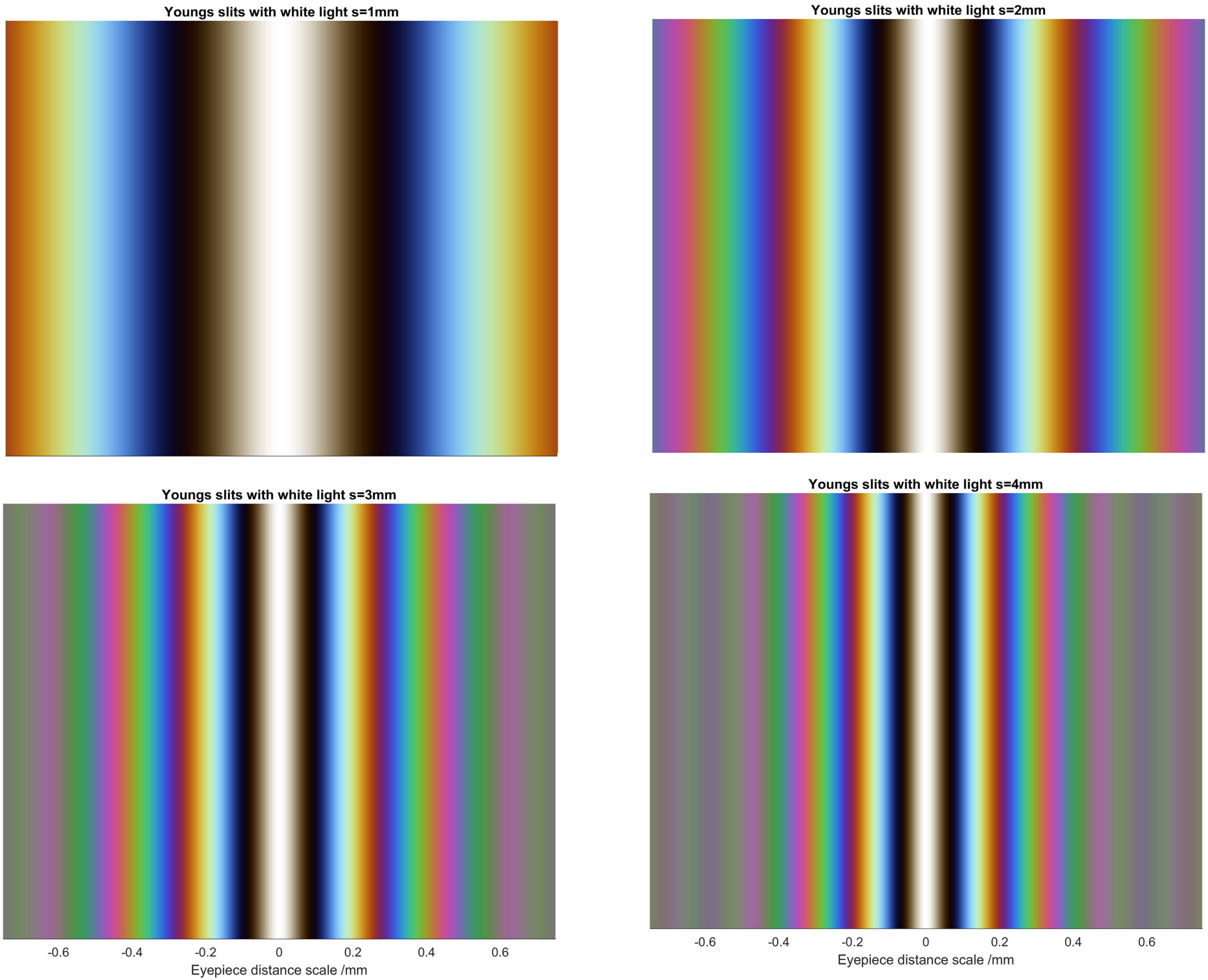
```
%Far-field diffraction pattern of N slits of width w (mm)
%and spacing s (mm). Note intensity I is normalized to unity
%at theta=0. wavelength in nm and angle theta from boresight
%in radians.
function I = fraunhofer(theta,lamda,w,s,N)
lamda = 1e3*lamda*1e-9; %Convert wavelength to mm
a = pi*w*sin(theta)/lamda;
b = pi*N*s*sin(theta)/lamda;
c = pi*s*sin(theta)/lamda;
I = ( sin(a).*sin(b)./( a.*sin(c) ) ).^2 ;
```

Joseph von Fraunhofer
1787-1826

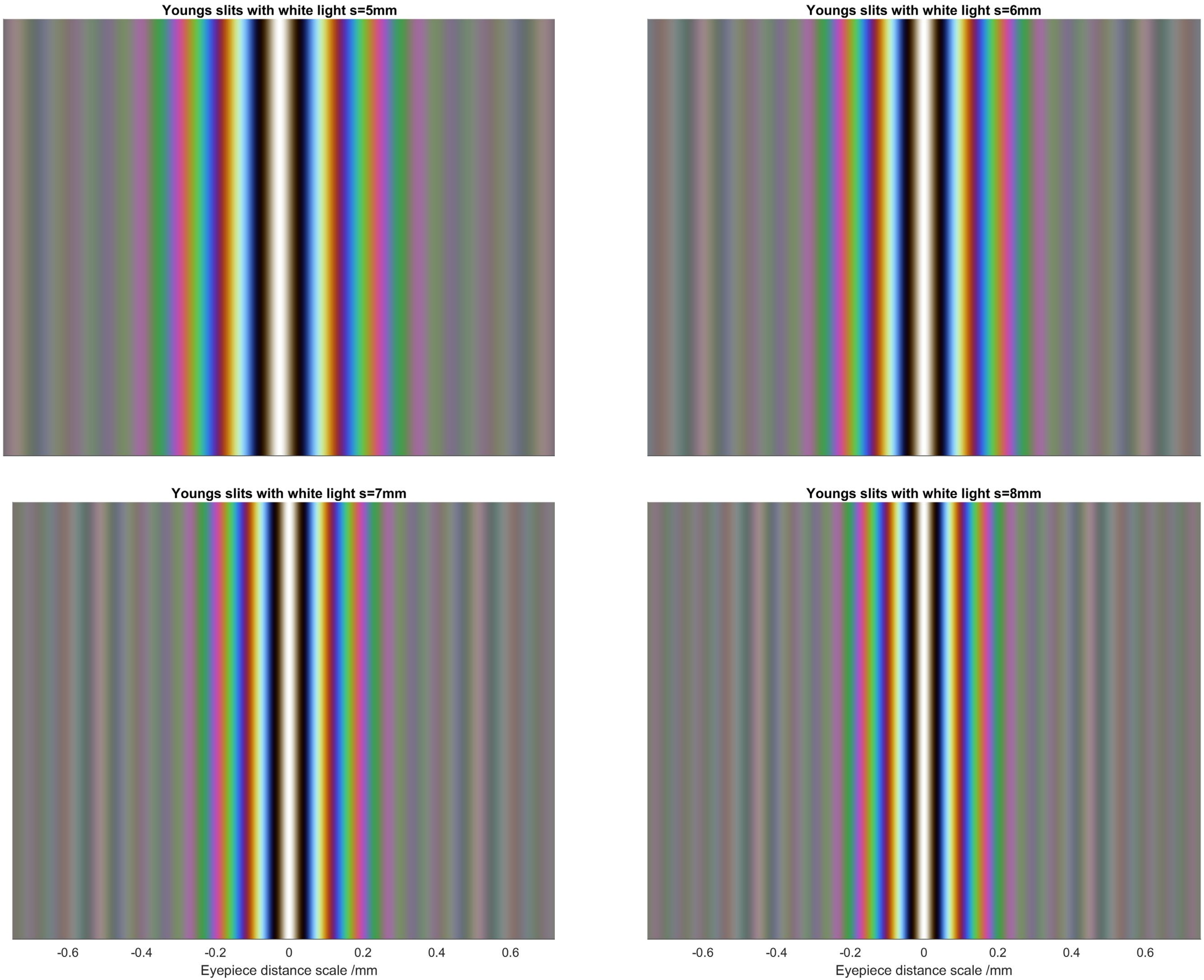
$$|\psi|^2 = \frac{A^2}{N^2 r^2}\left(\frac{\sin\left(\frac{\pi}{\lambda} w \sin\theta\right)}{\frac{\pi}{\lambda} w \sin\theta} \times \frac{\sin\left(\frac{\pi}{\lambda} N s \sin\theta\right)}{\sin\left(\frac{\pi}{\lambda} s \sin\theta\right)} \right)^2$$


Youngs slits with white light s=1mm, w=0.1mm
Youngs slits with white light s=2mm, w=0.1mm
Youngs slits with white light s=5mm, w=0.1mm
Youngs slits with white light s=10mm, w=0.1mm
Normalized intensity
Eyepiece distance scale /mm
0
0.5
1
1.5
2
2.5
3
3.5
4
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8

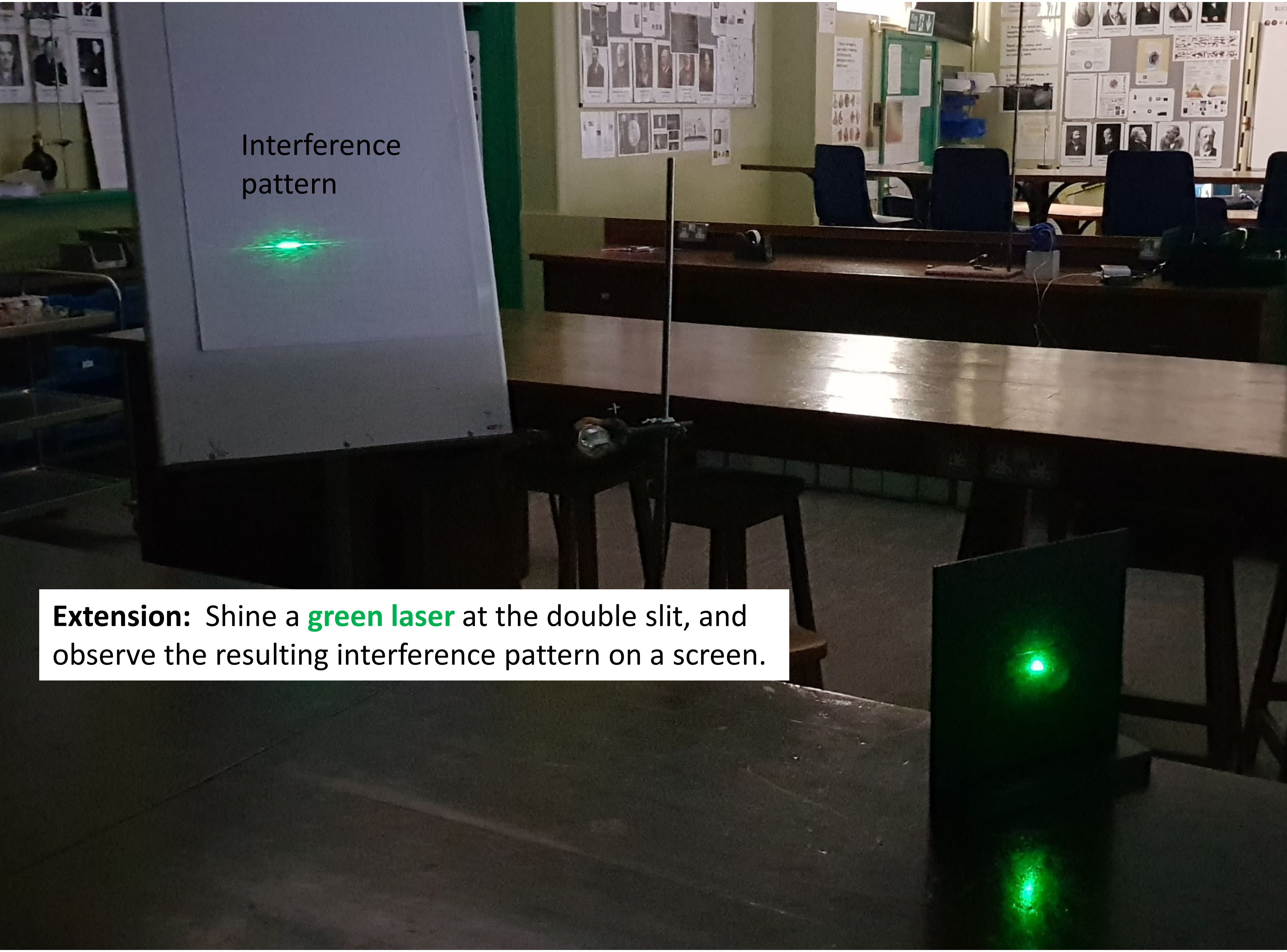
```
%Initialize R,G,B colours for each position along eyepiece
scale
R = zeros(1,P); G = zeros(1,P); B = zeros(1,P);

%Plot far-field pattern
thetamax = atan(0.5*xw/(1000*D));
theta = linspace( -thetamax,thetamax,P );
x = 1000*D*sin(theta);
axes('nextplot','add');
for n=1:length(lamda)
%Compute far-field intensity pattern
    I = fraunhofer(theta,lamda(n),w,s,2);
    RGB = RGB_from_f( f(n) );
    plot( x,I,'color', RGB );  %Overlay pattern
    %Add to total colour intensity,
    %weighted by far-field     pattern
    R = R + I*RGB(1); G = G + I*RGB(2); B = B + I*RGB(3);
End
%Normalize colour values
R = R/max(R); G = G/max(G); B = B/max(B);
```



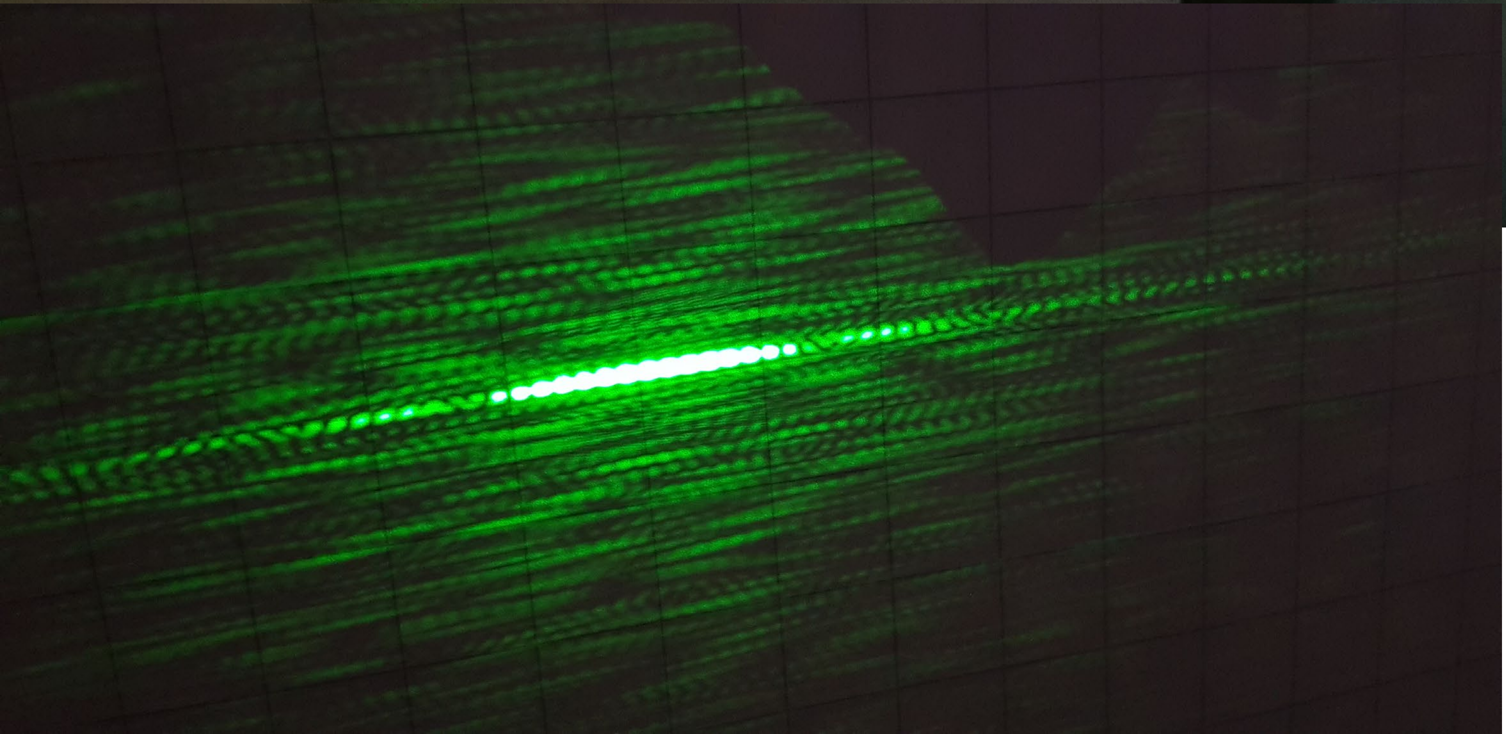
Youngs slits with white light s=1mm
Youngs slits with white light s=2mm
Youngs slits with white light s=3mm
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
Eyepiece distance scale /mm
Youngs slits with white light s=4mm
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
Eyepiece distance scale /mm


Youngs slits with white light s=5mm
Youngs slits with white light s=6mm
Youngs slits with white light s=7mm
Youngs slits with white light s=8mm
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
Eyepiece distance scale /mm
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
Eyepiece distance scale /mm

Extension: Shine a **green laser** at the double slit, and observe the Resulting interference pattern on a screen.



Extension: Shine a **green laser** at the double slit, and observe the resulting interference pattern on a screen.



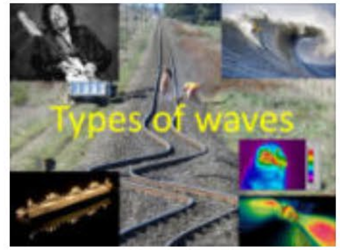
Note the detailed structure of maxima and minima since

$$s \approx 10,000\lambda$$

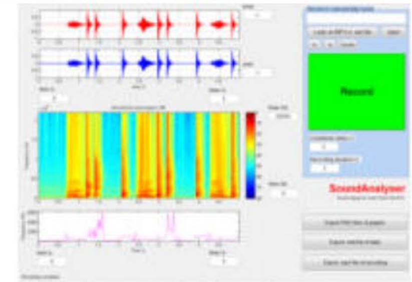
Watch out for backscatter – an interference pattern not an alien infection!

Waves & Optics

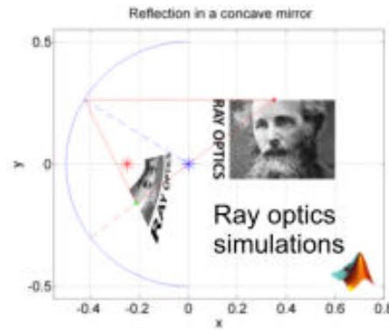
Mathematical anatomy of waves
Reflection & refraction
Diffraction Essentials
Diffraction (more advanced)
Standing waves
Doppler effect
EM waves, Fresnel equations & Polarisation
Geometric optics eclipses & lenses
Thin film interference



Instrument waveforms



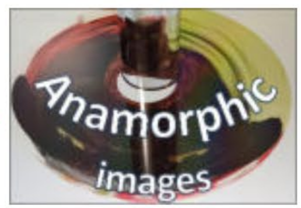
Waveform analysis software



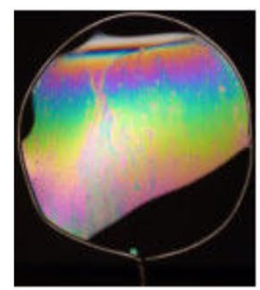
Doppler or Mach Excel

xls

MATLAB code
Double slit interference
Fraunhofer diffraction
Fresnel diffraction
Huygen's principle



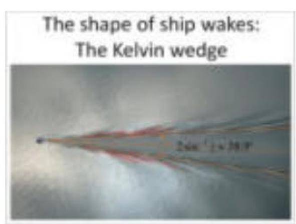
Junior part waves demonstration circus
Snell's Law practical & Excel model
Waves worksheet
EM spectrum cards (just pictures)



Soap film film

- Rubens Tube & cello
- Rubens Tube & theremin
- Speed of light in coax
- Speed of sound in air
- Perspex birefringence
- Fibre optic data link
- Laser diffraction
- Concave mirror

The Subtlety of Rainbows



Physics notes

Dr French's Eclecticon

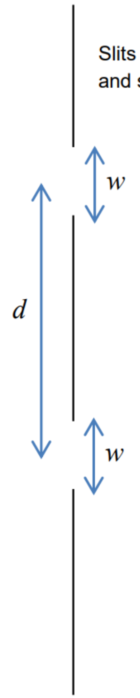
We shall model this by adding up the effect of an infinite number of thin slits, which make up the slit. This requires some Calculus, which we will not do here (see the full Diffraction note).

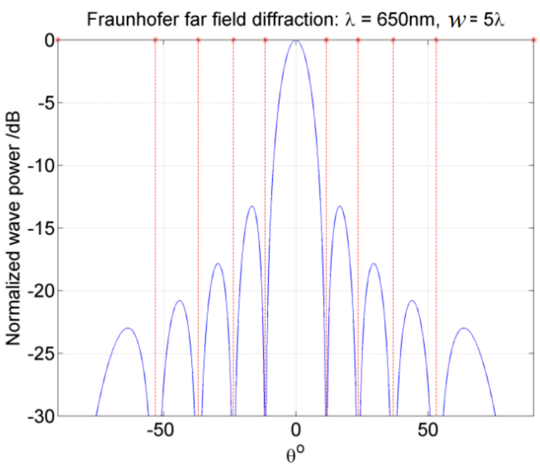
The end result is a envelope to the diffraction pattern which has **zeros** at

$$\sin\theta = \frac{m\lambda}{w}$$

m is any integer
w is the slit width

Slits of width w and spacing d





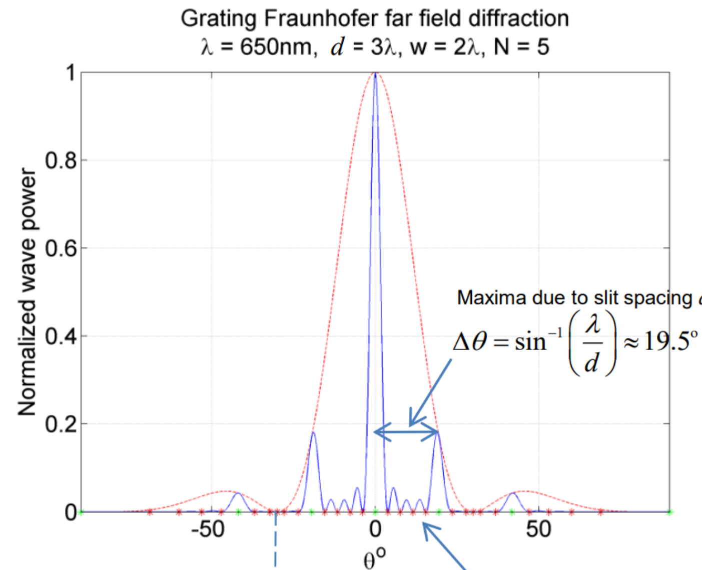
There are N slits (i.e. not just one or two...)

This will result in a **fine structure** (i.e. lots of extra little maxima). The maxima due to the slit spacing will appear *sharper*, and their will be *additional zeros* when

$$\sin\theta = \frac{p\lambda}{Nd}$$

p is any integer
N slits of slit width d

Caveat: there is a **maximum** when p/N is an integer i.e. angles corresponding to the maxima due to the slit spacing.



$$\Delta\theta = \sin^{-1}\left(\frac{\lambda}{d}\right) \approx 19.5^{\circ}$$

$$\Delta\theta = \sin^{-1}\left(\frac{\lambda}{w}\right) = 30^{\circ}$$

Minima of **envelope** due to finite slit width w

Fine structure due to overall size of aperture, which comprises of $N = 5$ slits in this case.
The extra **minima** (see the red stars) are at

$$\theta = \sin^{-1}\left(\frac{p\lambda}{Nd}\right)$$

... but are **maxima** (green stars) when p/N is an integer.

Joseph von Fraunhofer
1787-1826

Far-Field diffraction summary

This is the actual formula for the **diffraction pattern wave power**. It incorporates all the maxima and minima effects described above.

$$|\psi|^2 = \frac{A^2}{N^2 r^2}\left(\frac{\sin\left(\frac{\pi}{\lambda} w \sin\theta\right)}{\frac{\pi}{\lambda} w \sin\theta} \times \frac{\sin\left(\frac{\pi}{\lambda} N s \sin\theta\right)}{\sin\left(\frac{\pi}{\lambda} s \sin\theta\right)}\right)^2$$

n, m, p are integers

Envelope due to finite slit width

Zeros at: $\theta = \sin^{-1}\left(\frac{n\lambda}{w}\right);\ n \neq 0$

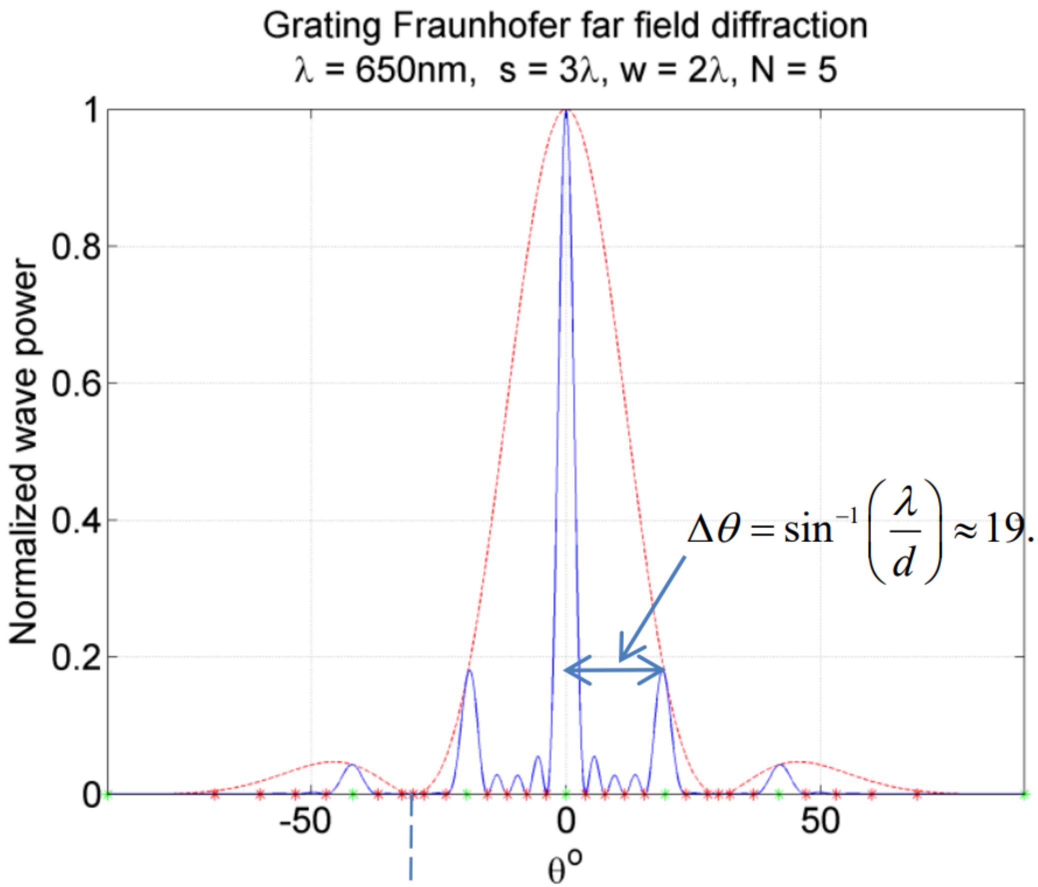
Maxima due to slit spacing

Maxima at: $\theta = \sin^{-1}\left(\frac{m\lambda}{s}\right)$

Fine structure due to number of slits
(i.e. overall size of aperture)

Zeros at: $\theta = \sin^{-1}\left(\frac{p\lambda}{Ns}\right)$

But *maxima* when $\frac{p}{N}$ integer m



$$\Delta\theta = \sin^{-1}\left(\frac{\lambda}{d}\right) \approx 19.5^\circ$$

$$\Delta\theta = \sin^{-1}\left(\frac{\lambda}{w}\right) = 30^\circ$$

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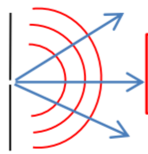
Diffraction Wavenumber $k=\frac{2\pi}{\lambda}$ $\omega=2\pi f$ Frequency Wave speed $c=f\lambda$ $\omega=ck$ Wave power input $P=\frac{1}{2}ZA^2\omega^2$ Z = Wave impedance

The *Huygens-Fresnel Principle* states: *"Every unobstructed point of a wavefront, at a given instant, serves as a source of spherical secondary wavelets (with the same frequency as that of the primary wave). The amplitude of the optical field at any point beyond is the superposition of all these wavelets (considering their amplitudes and relative phases)."*

Hence to determine the *wavefield* beyond an illuminated edge of slit, we need to add up the effect of spherical wave sources in the vicinity of the slit or *aperture*.

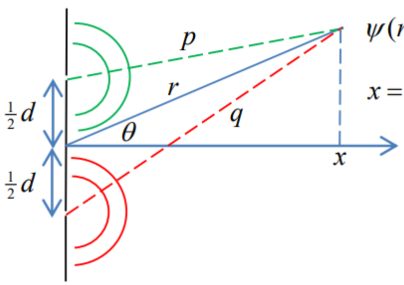
An infinitesimally thin slit



$$\psi(r,t)=\frac{Ae^{i(kr-\omega t)}}{r}$$

Two infinitesimally thin slits

'Young's double slits'



$$\psi(r,t)=\frac{Ae^{i(kp-\omega t)}}{p}+\frac{Ae^{i(kq-\omega t)}}{q}$$

$$x=r\cos\theta$$

Cosine Rule:

$$p^2=r^2+\tfrac{1}{4}d^2-rd\cos(90^\circ-\theta)$$

$$p^2=r^2+\tfrac{1}{4}d^2-rd\sin\theta$$

$$q^2=r^2+\tfrac{1}{4}d^2-rd\cos(90^\circ+\theta)$$

$$p^2=r^2+\tfrac{1}{4}d^2+rd\sin\theta$$

$$p=r\sqrt{1+\tfrac{1}{4}\frac{d^2}{r^2}-\frac{d\sin\theta}{r}}$$

$$q=r\sqrt{1+\tfrac{1}{4}\frac{d^2}{r^2}+\frac{d\sin\theta}{r}}$$

Assume $r\gg d$

$$\therefore\psi(r,t)\approx\frac{Ae^{-i\omega t}}{r}\left(e^{ikp}+e^{ikq}\right)$$

Only the phase term will vary significantly when $r\gg d$

Binomial expansion:

$$p\approx r+\tfrac{1}{8}\frac{d^2}{r}-\tfrac{1}{2}d\sin\theta$$

$$q\approx r+\tfrac{1}{8}\frac{d^2}{r}+\tfrac{1}{2}d\sin\theta$$

$$\therefore\psi(r,t)\approx\frac{Ae^{-i\omega t}}{r}e^{ik\left(r+\frac{1}{8}\frac{d^2}{r}\right)}\left(e^{-i\frac{1}{2}kd\sin\theta}+e^{i\frac{1}{2}kd\sin\theta}\right)$$

$$\psi(r,t)\approx\frac{2Ae^{-i\omega t}}{r}e^{ik\left(r+\frac{1}{8}\frac{d^2}{r}\right)}\cos\left(\tfrac{1}{2}kd\sin\theta\right)$$

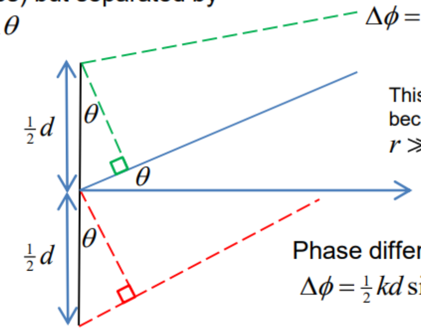
$$|\psi|^2\approx\frac{A^2}{r^2}\cos^2\left(\tfrac{1}{2}kd\sin\theta\right)$$ wave **power**

Hence *maxima* when

$\frac{1}{2}kd\sin\theta=n\pi$ n is an integer

$$\theta=\sin^{-1}\left(\frac{2n\pi}{kd}\right)=\sin^{-1}\left(\frac{n\lambda}{d}\right)$$ $$|n|<\frac{d}{\lambda}$$

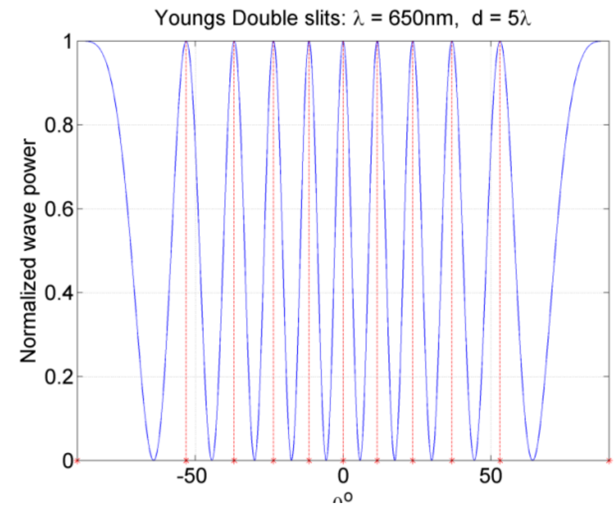
Equivalent geometry is wavelet sources are at the same angle (i.e. plane waves) but separated by $d\sin\theta$



Phase difference $\Delta\phi=-\frac{1}{2}kd\sin\theta$

This only works because $r\gg d$

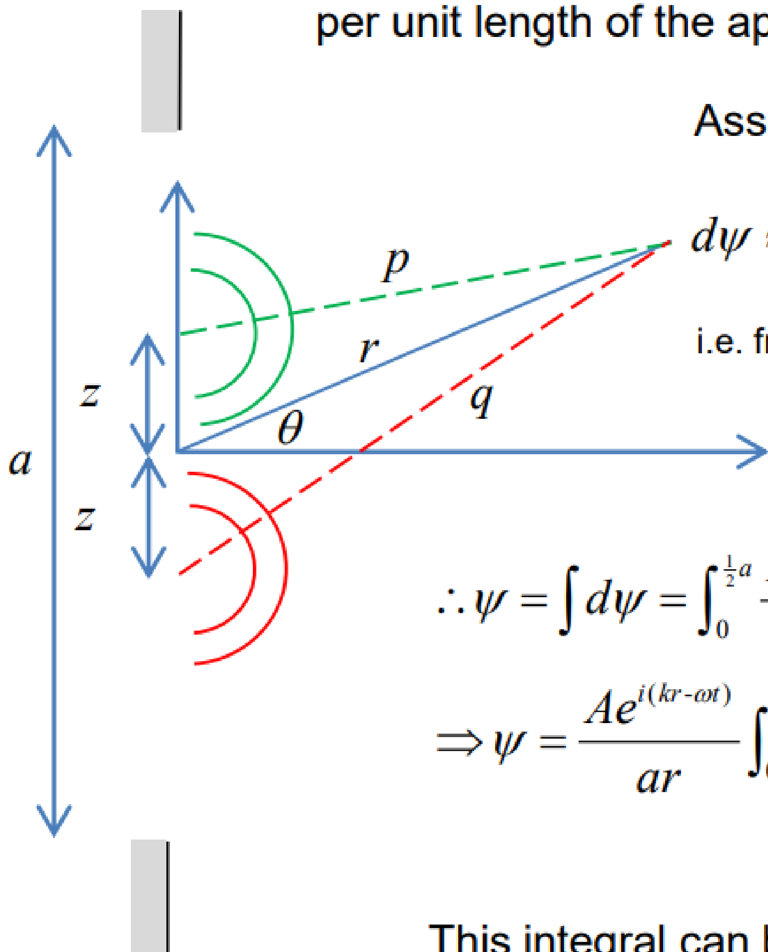
Phase difference $\Delta\phi=\frac{1}{2}kd\sin\theta$



The diffraction pattern of a finite width slit

The analysis of the double slit can be extended to include pairs of infinitesimal slits which cover the whole aperture width *a*

Define A/a in this case to be the illumination amplitude per unit length of the aperture



Assuming $r \gg a$

$$d\psi \approx \frac{A}{a} dz \frac{e^{-i\omega t}}{r} e^{ik\left(r+\frac{1}{2}\frac{z^2}{r}\right)} \left(e^{-ikz\sin\theta} + e^{ikz\sin\theta}\right)$$

i.e. from Double slit analysis, but use $\frac{1}{2}d \rightarrow z$

$$\therefore \psi = \int d\psi = \int_0^{\frac{1}{2}a} \frac{Ae^{-i\omega t}}{ar} e^{ik\left(r+\frac{1}{2}\frac{z^2}{r}\right)} \left(e^{-ikz\sin\theta} + e^{ikz\sin\theta}\right) dz$$

$$\Rightarrow \psi = \frac{Ae^{i(kr-\omega t)}}{ar} \int_0^{\frac{1}{2}a} e^{\frac{ikz^2}{2r}} \left(e^{-ikz\sin\theta} + e^{ikz\sin\theta}\right) dz$$

This integral can be simplified into two regimes:

Fraunhofer – or 'linear phase' with *z* $e^{\frac{ikz^2}{2r}} \approx \text{constant}$

$$\frac{k\left(\frac{1}{2}a\right)^2}{2r} \ll 1$$

$$\frac{2\pi a^2}{8\lambda r} \ll 1$$

$$r \gg \frac{\pi a^2}{4\lambda}$$

$$r \gg \frac{a^2}{\lambda}$$

$$\psi = \frac{Ae^{i(kr-\omega t)}}{ar}\int_0^{\frac{1}{2}a}\left(e^{-ikz\sin\theta} + e^{ikz\sin\theta}\right)dz$$

$$\psi = \frac{2Ae^{i(kr-\omega t)}}{ar}\int_0^{\frac{1}{2}a}\cos\left(kz\sin\theta\right)dz$$

$$\psi = \frac{2Ae^{i(kr-\omega t)}}{ar}\left[\frac{\sin\left(kz\sin\theta\right)}{k\sin\theta}\right]_0^{\frac{1}{2}a}$$

$$|\psi|^2 = \frac{A^2}{r^2}\left(\frac{\sin\left(\frac{1}{2}ka\sin\theta\right)}{\frac{1}{2}ka\sin\theta}\right)^2$$

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1787-1826

$$\theta = 0; \quad |\psi|^2 = |\psi_0|^2 = \frac{A^2}{r^2}$$

$$\therefore \left|\frac{\psi}{\psi_0}\right|^2 = \left(\frac{\sin\left(\frac{1}{2}ka\sin\theta\right)}{\frac{1}{2}ka\sin\theta}\right)^2$$

Hence *zeros* when:

$\frac{1}{2}kd\sin\theta = n\pi$ *n* is a non-zero integer

$$\theta = \sin^{-1}\left(\frac{2n\pi}{kd}\right) \qquad |n| < \frac{kd}{2\pi}$$